Physics 522: Quantum Mechanics II

Lecture 23 Introduction to perturbation theory: Magnetic Resonance

We have focused this semester on kinematics, the structure of matter as described by the energy levels. We know want to focus of dynamics, the time evolution of quantum system. In particular, we are interested in dynamics generated when a system is driven by an external field, e.g. absorption or emission of light by an atom. Of central importance is the phenomenon of resonance, whereby a the external force oscillates at some frequency that is close to a natural oscillation frequency of system oscillated with its binding forces. The amazing thing about resonance is that even when the applied force is tiny compared to the binding force, it can have profound dynamical effect on the system when the applied frequency is close to the binding frequency (think Tacoma bridge). Our goal is to understand resonance in quantum mechanics.

The starting by in time-dependent perturation theory. The generic form of the Hamiltonian is

$$\hat{H}(t) = \hat{H}_0 + \hat{H}_{int}(t)$$

Where $\hat{H}_0$ defines the energy levels of system, and $\hat{H}_{int}(t)$ describes its interaction with an externally applied force. The interaction is explicitly time-dependent because it is a functional of a time dependent parameter, e.g. an oscillating electromagnetic field that we treat classically. As in time-independent perturbation theory, $\hat{H}_{int}(t)$ is assumed to be "small" compared to $\hat{H}_0$. Nonetheless, it can have profound effect on the dynamics due to resonance. In particular, if $\hat{H}_0$ has two energy levels $|1\rangle, |2\rangle$, with Bohr frequency $\omega_{12} = \frac{E_1 - E_2}{\hbar}$ associated with the energy-level difference,

then when $\hat{H}_{int}(t)$ oscillates as a frequency ω close to ω_{12}, it have drive resonant absorption and emission between the levels.

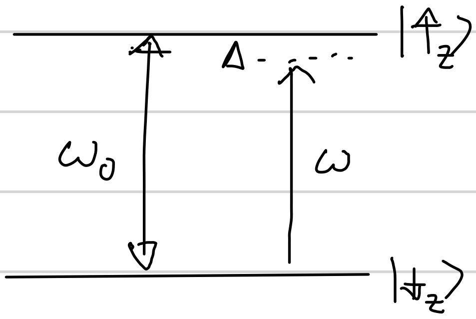
Magnetic Resonance

A key paradigm that exemplifies the phenomenon of resonance in quantum mechanics is magnetic resonance.

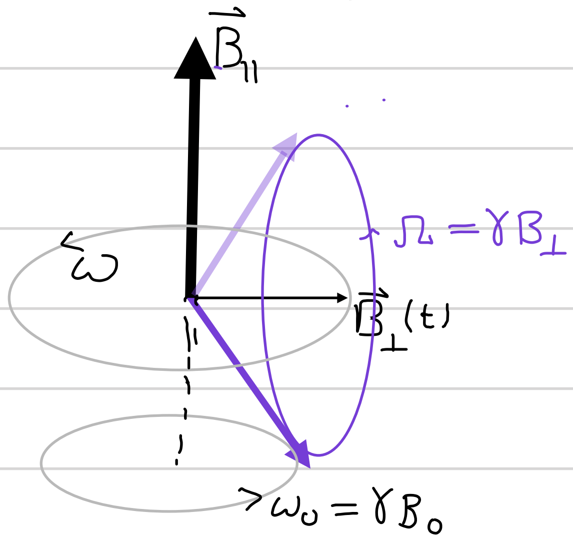
- Apply a strong magnetic field $\vec{B}_0$ along some axis: Defines the "quantization axis" z: $\vec{B}_0 \equiv B_{\parallel}\vec{e}_z$

$$\hat{H}_0 = -\hat{\mu}\cdot\vec{B}_0 = \frac{\hbar\omega_0}{2}\hat{\sigma}_z \qquad \omega_0 = |\gamma| B_0$$

- "Drive" the system with time dependent interaction, $\vec{B}_{int}(t)$, oscillating near resonance, ω near ω_0, $\hat{H}_{int} = -\hat{\vec{\mu}}\cdot\vec{B}_{int}(t)$



To drive the spin from $|\downarrow_z\rangle \Rightarrow |\uparrow_z\rangle$, the perturbing Hamiltonian must have off-diagonal matrix elements $\Rightarrow \hat{H}_{int}$ must have term $\propto \hat{\sigma}_x$ and/or $\hat{\sigma}_y \Rightarrow \vec{B}_{int}(t)$ in x-y plane. To achieve resonance, consider the following geometry:



In the presence of the static field $\vec{B}_{\parallel}$, the spin precesses at freq $\omega_0 = |\gamma| B_{\parallel}$. By applying a transverse field that rotates in the x-y plane, we can achieve perfect resonance, flipping the spin from $|\downarrow_z\rangle$ to $|\uparrow_z\rangle$.

Mathematically, we seek the solution to the time-dependent Schrödinger equation:

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -\frac{i}{\hbar}\hat{H}(t)|\psi(t)\rangle, \quad \hat{H}(t) = \hat{H}_0 + \hat{H}_{int}(t),$$

$$\hat{H}_0 = -\hat{\vec{\mu}}\cdot\vec{B}_{\parallel} = \frac{\hbar\omega_0}{2}\hat{\sigma}_z, \quad \hat{H}_{int} = -\vec{\mu}\cdot\vec{B}_{\perp}(t) = -\frac{\hbar\gamma B_x(t)}{2}\hat{\sigma}_x - \frac{\hbar\gamma B_y(t)}{2}\hat{\sigma}_y$$

For the specific case of a rotating transverse field of constant amplitude:
$\vec{B}_{\perp}(t) = B_{\perp}(\cos(\omega t+\phi)\vec{e}_x + \sin(\omega t+\phi)\vec{e}_y) \Rightarrow$

$$\hat{H}_{int}(t) = -\frac{\hbar\gamma B_{\perp}}{2}(\cos(\omega t+\phi)\hat{\sigma}_x + \sin(\omega t+\phi)\hat{\sigma}_y) = \frac{\hbar\Omega}{2}\Big(\underbrace{e^{-i(\omega t+\phi)}\hat{\sigma}_+}_{\substack{\hat{H}^{(+)} \\ \text{(positive freq)}}} + \underbrace{e^{i(\omega t+\phi)}\hat{\sigma}_-}_{\substack{\hat{H}^{(-)} \\ \text{(negative freq.)}}}\Big)$$

$\equiv \Omega$ the "Rabi frequency" $= -\gamma B_1$ (taking $\gamma < 0$)

From the geometrical picture, we see that if we go to a "rotating frame" co-rotating with the rotating field $\vec{B}_{\perp}(t)$, then in that frame, the Hamiltonian is static, and we can trivially integrate the Schrödinger equation.

Going to the rotating frame

We accomplish a frame transformation in quantum mechanics by making a unitary transformation. In the case of magnetic spin resonance, this is a physical rotation; in other cases (as we will see) the frame is abstract, and going to a rotating frame just means shifting the eigenvalues of the Hamiltonian, e.g. the familiar "interaction picture" in time-dependent perturbations.

Here, we move to rotating frame by rotation about the z-axis by angle ωt, where ω is the frequency of the rotating field $\vec{B}_{\perp}(t)$: $\hat{U}_{RF}(t) = e^{-i\frac{\omega t}{2}\hat{\sigma}_z}$

Observables in the rotating frame: $\hat{O}_{RF}(t) = \hat{U}^{\dagger}_{RF}(t)\,\hat{O}_S(t)\,\hat{U}_{RF}(t)$ Schröndinger Picture

States in the rotating frame: $|\psi_{RF}(t)\rangle = \hat{U}^{\dagger}_{RF}(t)|\psi_S(t)\rangle$, so $\langle\psi_S(t)|\hat{O}_S(t)|\psi_S(t)\rangle = \langle\psi_{RF}(t)|\hat{O}_{RF}(t)|\psi_{RF}(t)\rangle$.

Schrödinger Eqn in the rotating frame:

$$\frac{\hbar}{-i}\frac{\partial}{\partial t}|\psi_{RF}(t)\rangle = \frac{\hbar}{-i}\frac{\partial}{\partial t}\left[\hat{U}^{\dagger}_{RF}(t)\,|\psi_{s}(t)\rangle\right] = \hat{U}^{\dagger}_{RF}\left[\frac{\hbar}{-i}\frac{\partial}{\partial t}|\psi_{s}(t)\rangle\right] + \left[\frac{\hbar}{-i}\frac{\partial \hat{U}^{\dagger}_{RF}}{\partial t}\right]|\psi_{s}(t)\rangle$$

$$\Rightarrow \frac{\hbar}{-i}\frac{\partial}{\partial t}|\psi_{RF}(t)\rangle = \left[\hat{U}^{\dagger}_{RF}\hat{H}_{s}(t) + \frac{\hbar}{-i}\frac{\partial \hat{U}^{\dagger}_{RF}}{\partial t}\right]|\psi_{s}\rangle = \underbrace{\left[\hat{U}^{\dagger}_{RF}\hat{H}_{s}\hat{U}_{RF} + \frac{\hbar}{-i}\frac{\partial \hat{U}^{\dagger}_{RF}}{\partial t}\hat{U}_{RF}\right]}_{\hat{H}_{RF}}|\psi_{RF}\rangle$$

⇒ New Hamiltonian in the rotating frame

$$\hat{H}_{RF} = \hat{U}^{\dagger}_{RF}\hat{H}_{s}\hat{U}_{RF} - \frac{\hbar\omega}{2}\hat{\sigma}_{z}, \qquad \hat{H}_{s} = \frac{\hbar\omega_0}{2}\hat{\sigma}_{z} + \hbar\Omega\left(e^{-i(\omega t+\phi)}\hat{\sigma}_{+} + e^{i(\omega t+\phi)}\hat{\sigma}_{-}\right).$$

$$\Rightarrow \hat{H}_{RF} = -\frac{\hbar(\omega-\omega_0)}{2}\hat{\sigma}_{z} + \hbar\Omega\left(e^{-i(\omega t+\phi)}\hat{U}^{\dagger}_{RF}\hat{\sigma}_{+}\hat{U}_{RF} + e^{i(\omega t+\phi)}\hat{U}^{\dagger}_{RF}\hat{\sigma}_{-}\hat{U}_{RF}\right)$$

I leave it as a simple exercise to show: $U^{\dagger}_{RF}\hat{\sigma}_{\pm}U_{RF} = e^{\pm i\omega t}\hat{\sigma}_{\pm}$

$$\Rightarrow \hat{H}_{RF} = -\frac{\hbar\Delta}{2}\hat{\sigma}_{z} + \frac{\hbar\Omega}{2}\left(e^{-i\phi}\hat{\sigma}_{+} + e^{+i\phi}\hat{\sigma}_{-}\right) = -\frac{\hbar\Delta}{2}\hat{\sigma}_{z} + \frac{\hbar\Omega}{2}\left(\cos\phi\,\hat{\sigma}_{x} + \sin\phi\,\hat{\sigma}_{y}\right)$$

$\Delta = \omega - \omega_0$ (detuning), $\Omega = \gamma B_{\perp}$ (Rabi frequency)

$\hat{H}_{RF}$ is time-independent as expected!

$$\hat{H}_{RF} = \frac{\hbar\vec{\Omega}_{tot}}{2}\cdot\hat{\vec{\sigma}}, \qquad \vec{\Omega}_{tot} = -\Delta\vec{e}_{z} + \Omega\,\vec{e}_{\perp}(\phi) \quad (\vec{e}_{\perp} = \vec{e}_{x}\cos\phi + \vec{e}_{y}\sin\phi)$$

General solution: $|\psi_{RF}(t)\rangle = e^{-\frac{i}{\hbar}\hat{H}_{RF}t}|\psi_{RF}(0)\rangle = \hat{U}_{Rabi}(t)|\psi_{RF}(0)\rangle$

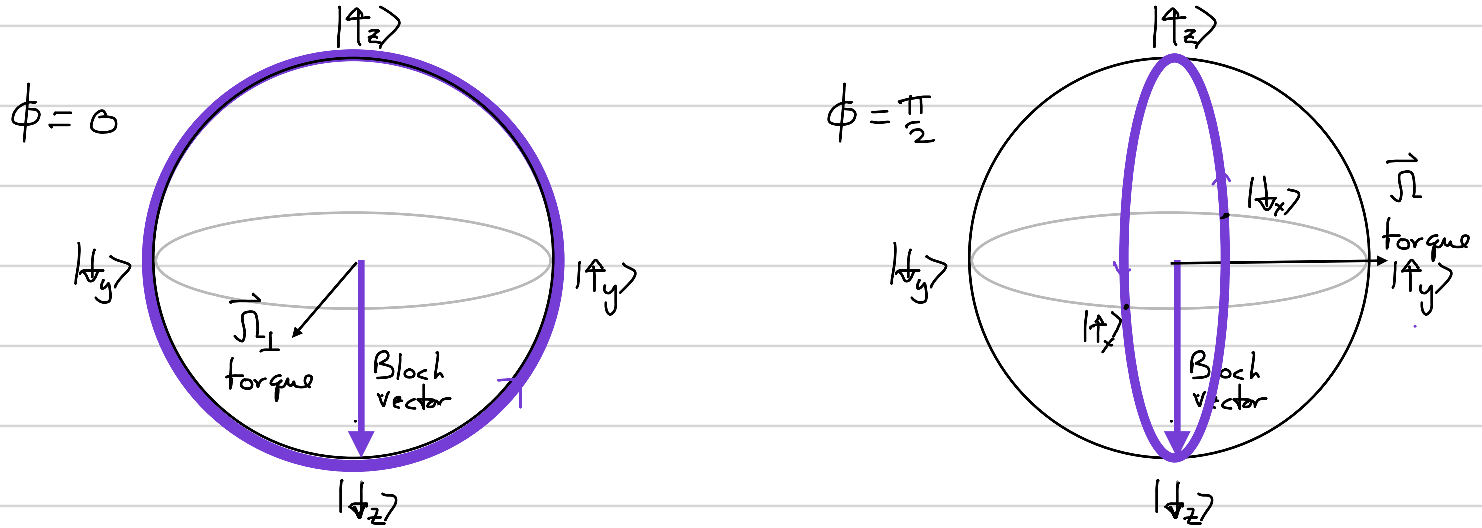
$\hat{U}_{Rabi} = e^{-i\frac{\vec{\Omega}_{tot}}{2}\cdot\hat{\vec{\sigma}}}$: Rotation on the Bloch sphere

"Generalized Rabi frequency": $\Omega_{tot} = |\vec{\Omega}_{tot}| = \sqrt{\Omega^2 + \Delta^2}$

axis of rotation $\vec{e}_{a} = \frac{\vec{\Omega}_{tot}}{|\vec{\Omega}_{tot}|} = -\frac{\Delta}{\Omega_{tot}}\vec{e}_{z} + \frac{\Omega}{\Omega_{tot}}\vec{e}_{\perp}(\phi)$

Consider the case $\Delta = 0$ (on resonance)

$$\hat{H}_{RF} = \frac{\hbar\Omega}{2}\vec{e}_\perp(\phi)\cdot\hat{\vec{\sigma}} = \frac{\hbar\Omega}{2}\left(\cos\phi\,\hat{\sigma}_x + \sin\phi\,\hat{\sigma}_y\right)$$



Rabi rotations on Bloch sphere.

On resonance the Bloch vector precesses from north to south pole about an axis depending on ϕ

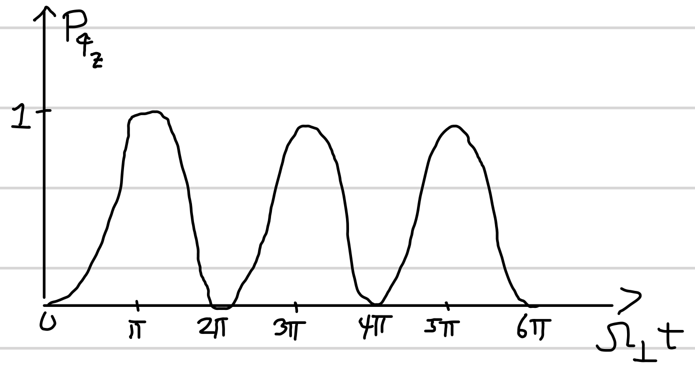
Written in terms of the Quantum evolution in the rotating frame: $|\psi_{RF}(t)\rangle = \hat{U}_{Rabi}|\downarrow_z\rangle$

$$\hat{U}_{Rabi} = e^{-i\vec{\Omega}_{tot}t\cdot\frac{\hat{\vec{\sigma}}}{2}} = \cos\left(\frac{\Omega_{tot}t}{2}\right)\hat{\mathbb{1}} - i\sin\left(\frac{\Omega_{tot}t}{2}\right)\vec{e}_a\cdot\hat{\vec{\sigma}}$$

$$= \cos\left(\frac{\Omega_{tot}t}{2}\right)\hat{\mathbb{1}} - i\sin\left(\frac{\Omega_{tot}t}{2}\right)\left[-\frac{\Delta}{\Omega_{tot}}\hat{\sigma}_z + \frac{\Omega}{\Omega_{tot}}\left(e^{-i\phi}\hat{\sigma}_+ + e^{+i\phi}\hat{\sigma}_-\right)\right]$$

On resonance $|\psi_{RF}(t)\rangle = \hat{U}_{Rabi}|\downarrow_z\rangle = \cos\left(\frac{\Omega_{tot}t}{2}\right)|\downarrow_z\rangle - i\,e^{-i\phi}\sin\left(\frac{\Omega_{tot}t}{2}\right)|\uparrow_z\rangle$

$$P_{\uparrow_z}(t) = |\langle\uparrow_z|\psi_{RF}(t)\rangle|^2 = \sin^2\left(\frac{\Omega t}{2}\right) = \frac{1+\cos(\Omega t)}{2}$$



Rabi flopping !

Population oscillates from $|\downarrow_z\rangle$ to $|\uparrow_z\rangle$.

Ex: "π-pulse", $\Omega_\perp t = \pi \Rightarrow |\psi_{RF}(\frac{\pi}{\Omega})\rangle = -ie^{i\phi}|\uparrow_z\rangle \equiv |\uparrow_z\rangle$

A π-pulse flips spin-down to spin-up. It represents "perfect absorption."
But this is not the full story. For suppose we stopped the pulse half-way, i.e $\Omega t = \frac{\pi}{2}$.

Ex: "$\frac{\pi}{2}$-pulse", $\Omega_\perp t = \frac{\pi}{2}$ $\Rightarrow$ $|\psi_{RF}(\frac{\pi}{2\Omega})\rangle = \frac{1}{\sqrt{2}}(|\downarrow_z\rangle - i e^{-i\phi}|\uparrow_z\rangle) = \frac{-ie^{-i\phi}}{\sqrt{2}}(|\uparrow_z\rangle + i e^{i\phi}|\downarrow_z\rangle)$

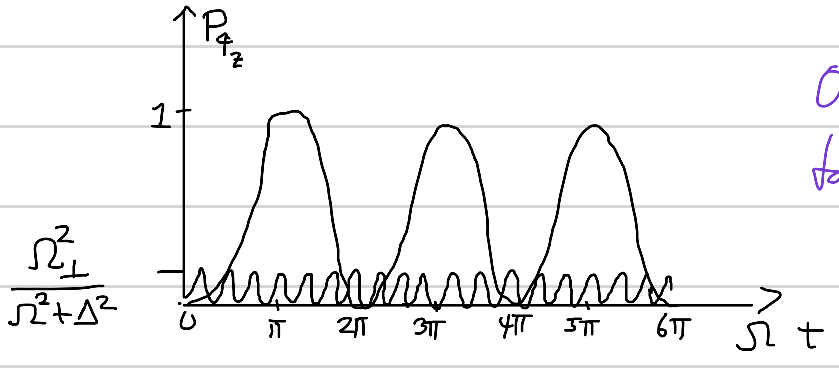
$\Rightarrow$ A $\frac{\pi}{2}$-pulse of magnetic energy acting on $|\downarrow_z\rangle$ creates a 50-50 superposition of $|\uparrow_z\rangle$ and $|\downarrow_z\rangle$ which a phase between them that depends on the phase of the applied oscillator.
An important point is that the evolution is coherent. That is, at all stages of the evolution, the spin is in a coherent superposition of $|\uparrow_z\rangle$ and $|\downarrow_z\rangle$. Stopping the coherent evolution half wave from spin-up to spin down leaves the system in a 50-50 superposition

Note: For a 2π-pulse, $|\psi(\frac{2\pi}{\Omega})\rangle = -|\downarrow_z\rangle$. The accumulation of the phase -1 has no physical effect on a spin-$\frac{1}{2}$ state. But it reflects the difference between SU(2) rotations and SO(3) rotations in Euclidean 3D space.

General Rabi Solution: $|\psi_{RF}(t)\rangle = \hat{U}_{Rabi}|\psi(0)\rangle$, with $|\psi(0)\rangle = |\downarrow_z\rangle$

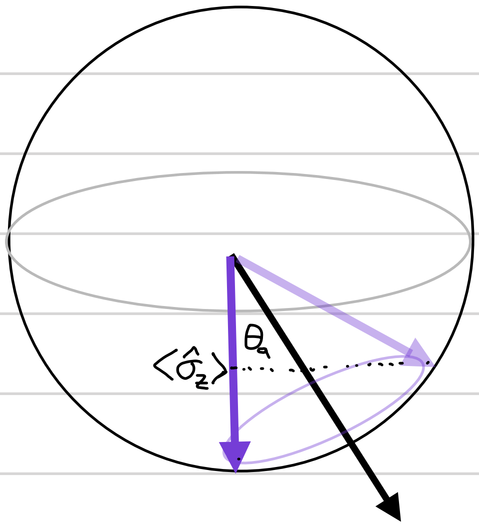
$$\Rightarrow |\psi_{RF}(t)\rangle = \left[\cos\left(\frac{\Omega_{tot}t}{2}\right) + i\frac{\Delta}{\Omega_{tot}}\sin\left(\frac{\Omega_{tot}t}{2}\right)\right]|\downarrow_z\rangle + \left[-ie^{-i\phi}\frac{\Omega}{\Omega_{tot}}\sin\left(\frac{\Omega_{tot}t}{2}\right)\right]|\uparrow_z\rangle$$

$$P_{\uparrow_z}(t) = |\langle\uparrow_z|\psi_{RF}(t)\rangle|^2 = \frac{\Omega^2}{\Omega_{tot}^2}\sin^2\left(\frac{\Omega_{tot}t}{2}\right) = \frac{\Omega^2}{\Omega^2+\Delta^2}\sin^2\left(\frac{\Omega_{tot}t}{2}\right)$$



Off resonance, there is never unit probability for the spin to go from $\downarrow_z \Rightarrow \uparrow_z$.
The probability amplitude also oscillates faster @ $\Omega_{tot} = \sqrt{\Omega^2+\Delta^2}$

Off-Resonance Bloch Sphere Picture



$$\vec{\Omega}_{tot} = -\Delta\vec{e}_z + \Omega\,\vec{e}_\perp$$

The angle the torque axis with the -z axis $\tan\theta_a = \frac{\Omega}{-\Delta}$

$$\Rightarrow \langle\hat{\sigma}_z\rangle^{max} = -\cos 2\theta_a = 2\sin^2\theta_a - 1$$

$$= 2\,\frac{\Omega^2}{\Omega^2+\Delta^2} - 1 = P_{\uparrow_z} - P_{\downarrow_z}$$

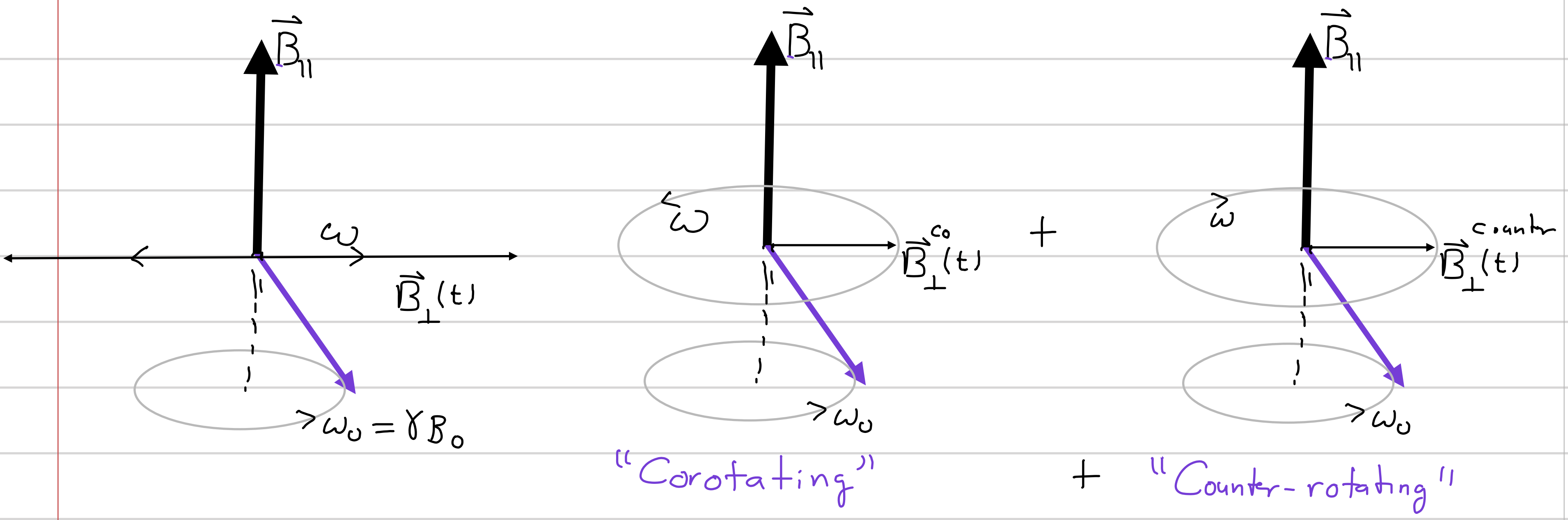
$$= 2P_{\uparrow_z} - 1 \Rightarrow P_{\uparrow_z} = \frac{\Omega_\perp^2}{\Omega_\perp^2+\Delta^2}$$

✓

Rotating wave approximation (RWA)

Suppose that instead of rotating transverse field, we had a linearly oscillating field along a transverse axis, say the x-axis

$$B_x\cos(\omega t)\,\vec{e}_x = \underbrace{\frac{B_x}{2}\left[\cos\omega t\,\vec{e}_x + \sin\omega t\,\vec{e}_y\right]}_{\text{Right hand circulating}} + \underbrace{\frac{B_x}{2}\left[\cos\omega t\,\vec{e}_x - \sin\omega t\,\vec{e}_y\right]}_{\text{Left hand circulating}}$$



The linearly oscillating field decomposes into "co-rotating" and "counter-rotating" terms. Only the co-rotating term in near resonance. When $|\omega-\omega_0| \ll \omega_0$ and $\Omega \ll \omega_0$, the counter rotating term oscillates so fast in the rotating frame, that " net effect on the Bloch vector is negligible. This is known as the rotating wave approximation (RWA).

Formally, consider the interaction Hamiltonian in the Schrödinger picture

$$\hat{H}_{int}^{(s)} = -\vec{\mu}\cdot\vec{e}_x B_x \cos\omega t = \frac{-\hbar\gamma}{2} B_x \cos\omega t\, \hat{\sigma}_x = \frac{-\hbar\gamma B_x}{4}\left(e^{-i\omega t} + e^{+i\omega t}\right)(\hat{\sigma}_+ + \hat{\sigma}_-)$$

Transforming to the rotating frame

$$\hat{H}_{int}^{(RF)} = \frac{-\hbar\gamma B_x}{4}\left(e^{-i\omega t} + e^{+i\omega t}\right)\left(\hat{\sigma}_+ e^{+i\omega t} + \hat{\sigma}_- e^{-i\omega t}\right)$$

$$= \underbrace{\frac{-\hbar\gamma B}{4}(\hat{\sigma}_+ + \hat{\sigma}_-)}_{\text{Co-rotating terms}} \underbrace{- \frac{\hbar\gamma B_x}{4}\left(e^{+2i\omega t}\hat{\sigma}_+ + e^{-2i\omega t}\hat{\sigma}_-\right)}_{\text{Counter-rotating terms}} \approx \frac{\hbar\Omega}{4}(\hat{\sigma}_+ + \hat{\sigma}_-)$$

$$\Omega = -\gamma B_x$$

The counter-rotating terms oscillate like 2ω, Whereas the characteristic dynamics of the Bloch vector is at rates $\Omega, \Delta \Rightarrow$ Rapid oscillations average to zero
This can be made more rigorous using the "method of averages"

<u>Different Representations</u>

When examining the problem of Rabi oscillations, there are a number of different representations

- <u>Probability amplitudes</u>

In the rotating frame $|\psi_{RF}\rangle = c_{\uparrow_z}|\uparrow_z\rangle + c_{\downarrow_z}|\downarrow_z\rangle$

$$\hat{H}_{RF} = -\frac{\hbar\Delta}{2}\hat{\sigma}_z + \frac{\hbar\Omega}{2}\hat{\sigma}_x \quad (\text{choosing drive phase } \phi = 0)$$

$\Rightarrow$ Matrix representation

$$\underbrace{\frac{d}{dt}\begin{bmatrix} c_\uparrow \\ c_\downarrow \end{bmatrix}}_{|\psi_{RF}\rangle} = \underbrace{\frac{-i}{2}\begin{bmatrix} -\Delta & \Omega_1 \\ \Omega_1 & \Delta \end{bmatrix}}_{-\frac{i}{\hbar}\hat{H}_{RF}} \underbrace{\begin{bmatrix} c_\uparrow \\ c_\downarrow \end{bmatrix}}_{|\psi_{RF}\rangle} \Rightarrow \begin{aligned} \dot{c}_\uparrow &= \tfrac{i}{2}\Delta c_\uparrow - \tfrac{i}{2}\Omega c_\downarrow \\ \dot{c}_\downarrow &= -\tfrac{i}{2}\Delta c_\downarrow - \tfrac{i}{2}\Omega c_\uparrow \end{aligned}$$

On resonance: $\dot{c}_\uparrow = -i\frac{\Omega}{2}c_\downarrow$, $\dot{c}_\downarrow = i\frac{\Omega}{2}c_\uparrow \Rightarrow \ddot{c}_\uparrow = -\frac{\Omega^2}{4}c_\uparrow$ (SHO diff 'eqn)

$$\Rightarrow c_\uparrow(t) = c_\uparrow(0)\cos\left(\frac{\Omega t}{2}\right) + \frac{2}{\Omega}\dot{c}_\uparrow(0)\sin\left(\frac{\Omega t}{2}\right) = c_\uparrow(0)\cos\left(\frac{\Omega t}{2}\right) - i c_\downarrow(0)\sin\left(\frac{\Omega t}{2}\right)$$

with $c_\uparrow(0) = 0$, $c_\downarrow(0) = 1 \Rightarrow c_\uparrow(t) = -i\sin\left(\frac{\Omega t}{2}\right)$, $P_\uparrow(t) = \sin^2\left(\frac{\Omega t}{2}\right)$ ✓

Connection to absorption & emission of light by a two level atom

Our initial motivation to study two-level quantum systems was to study absorption and emission of light by atoms close to resonance.

The field drives transitions between two (nondegenerate) levels $|g\rangle \Rightarrow |\downarrow_z\rangle$ and $|e\rangle \Rightarrow |\uparrow_z\rangle$. The monochromatic field at the position of the atom is $\vec{E}(\vec{R},t) = \mathrm{Re}\left(\vec{E}_0\, e^{i\phi(\vec{R})}\, e^{-i\omega_L t}\right)$. The total Hamiltonian

$$\hat{H} = \hat{H}_A + \hat{H}_{AL_{\vec{R}}}(t)$$

Atom Hamiltonian

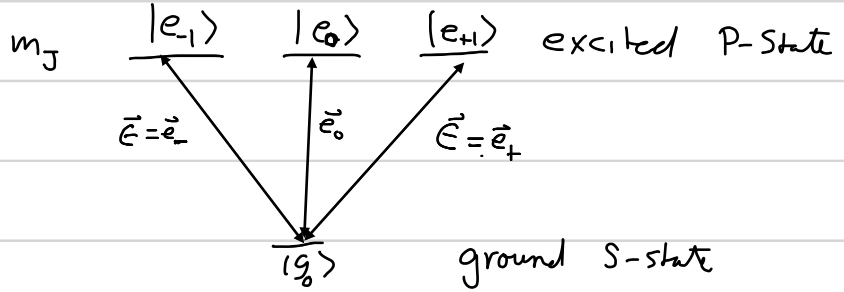
Atom-laser interaction Hamiltonian

$$\hat{H}_A = E_g|g\rangle\langle g| + E_e|e\rangle\langle e| = \frac{E_g+E_e}{2}\hat{\mathbb{1}} + \frac{E_e-E_g}{2}\hat{\sigma}_z \equiv \frac{\hbar\omega_{eg}}{2}\hat{\sigma}_z \,, \quad \omega_{eg} = \frac{E_e-E_g}{\hbar} \text{ (Bohr frequency).}$$

We take here $E_g + E_e = 0$

In the dipole approximation: $\hat{H}_{AL} = -\hat{\vec{d}}\cdot\vec{E}(\vec{R},t) = -\frac{\hat{\vec{d}}\cdot\vec{\epsilon}}{2}E_0\, e^{i\phi(\vec{R})}\, e^{-i\omega_L t} - \frac{\hat{\vec{d}}\cdot\vec{\epsilon}^{\,*}}{2}E_0\, e^{-i\phi(\vec{R})}\, e^{+i\omega_L t}$.

For concreteness, consider a dipole-allowed S→P atomic transition



According to the dipole-selection rules, $\Delta m_\ell = 0, \pm 1$. Thus, by choosing the polarization of the laser, we pick a given two-levels.

Consider the $\Delta m_e = 0$ transition (linear polarization along the quantization axis) so $\vec{\mathcal{E}}$ real

$$\hat{\vec{d}}\cdot\vec{\mathcal{E}} = |e\rangle\langle g|\,\langle e|\hat{\vec{d}}\cdot\vec{\mathcal{E}}|g\rangle + |g\rangle\langle e|\,\langle g|\hat{\vec{d}}\cdot\vec{\mathcal{E}}|e\rangle = d_{eg}\,(\hat{\sigma}_+ + \hat{\sigma}_-)$$

where $d_{eg} \equiv \langle e|\hat{\vec{d}}\cdot\vec{\mathcal{E}}|g\rangle$ is the dipole transition matrix element, which can be chosen real.

$$\Rightarrow \hat{H}_{AL} = \frac{-d_{eg}E_0}{2}(\hat{\sigma}_+ + \hat{\sigma}_-)\left(e^{i\phi}e^{-i\omega_L t} + e^{-i\phi}e^{i\omega_L t}\right)$$

This has the form of a magnetic spin resonance interaction: $\hat{H}_{int} = \hbar\Omega_\perp\,\hat{\sigma}_x\cos(\omega_L t - \phi)$.

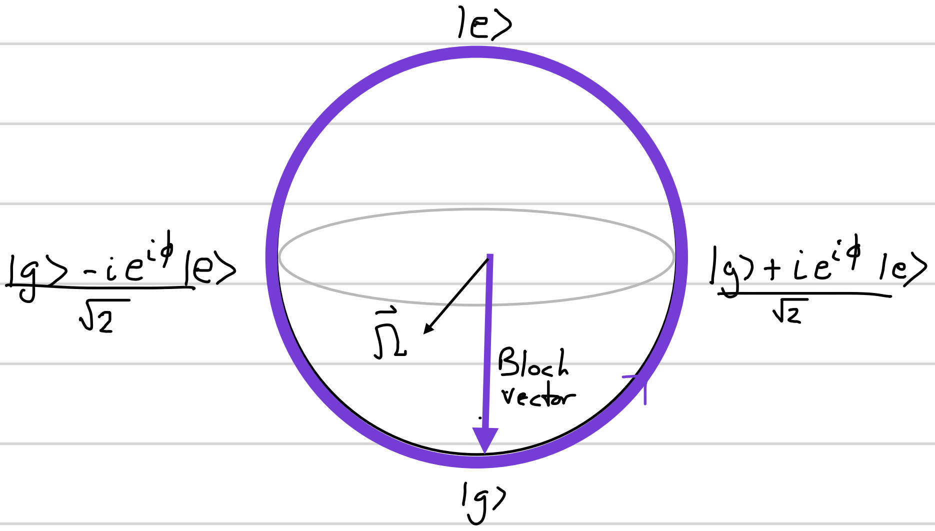
We thus define the Rabi frequency $\hbar\Omega = -d_{eg}E_0 = \langle e|\,e\hat{\vec{r}}\cdot\vec{\mathcal{E}}|g\rangle E_0$. We will drop the label $\perp$ from now on, when not talking about magnetic spin resonance. When $|\Delta| = |\omega_L - \omega_{eg}| << \omega_{eg}$ (near resonance) + $\Omega << \omega_{eg}$ we can make the rotating wave approximation:

In the RWA: $\hat{H}_{AL} = \frac{\hbar\Omega}{2}\left(\hat{\sigma}_+ e^{i\phi(\vec{R})}e^{-i\omega_L t} + \hat{\sigma}_- e^{-i\phi(\vec{R})}e^{i\omega_L t}\right)$. The Hamiltonian for absorption and emission thus has exactly the form of magnetic spin resonance!
Going to the rotating frame, having made the RWA

In rotating frame: $\hat{H} = -\frac{\hbar\Delta}{2}\hat{\sigma}_z + \frac{\hbar\Omega}{2}\left(\hat{\sigma}_+ e^{i\phi(\vec{R})} + \hat{\sigma}_- e^{-i\phi(\vec{R})}\right)$

$$\Delta = \omega_L - \omega_{eg}, \qquad \Omega = \frac{-d_{eg}E_0}{\hbar} = -\frac{\langle e|\hat{\vec{d}}\cdot\vec{\mathcal{E}}|g\rangle E_0}{\hbar}$$

The coherent interaction between the two-level atom and the laser field thus leads to Rabi flopping:



A $\frac{\pi}{2}$-pulse will create an atom is a coherent 50-50 superposition of $|g\rangle$ and $|e\rangle$