Physics 522, Spring 2016
Problem Set #2
Due: Tuesday Feb. 2, 2016 @ 5PM

Problem 1: 2D Isotropic SHO (20 points)

Consider again the 2D isotropic SHO described by the Hamiltonian

$$\hat{H} = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} + \frac{1}{2} m\omega^2 (\hat{x}^2 + \hat{y}^2)$$

(a) Show that $\hat{H} = \hbar\omega\left(\hat{a}_x^\dagger \hat{a}_x + \hat{a}_y^\dagger \hat{a}_y + 1\right) = \hbar\omega\left(\hat{N} + 1\right)$, where $\hat{a}_x = \frac{\hat{X} + i\hat{P}_x}{\sqrt{2}}$, $\hat{a}_y = \frac{\hat{Y} + i\hat{P}_y}{\sqrt{2}}$ (define dimensionless operators $\left(\hat{X}, \hat{Y}, \hat{P}_x, \hat{P}_y\right)$, and the total number operator $\hat{N} = \hat{a}_x^\dagger \hat{a}_x + \hat{a}_y^\dagger \hat{a}_y$.

(b) Show that the energy eigenvectors are

$$\left|n_x\right\rangle \otimes \left|n_y\right\rangle = \frac{\left(\hat{a}_x^\dagger\right)^{n_x} \left(\hat{a}_y^\dagger\right)^{n_y}}{\sqrt{n_x! n_y!}} \left|0_x\right\rangle \otimes \left|0_y\right\rangle$$

with energy eigenvalues are $E_n = \hbar\omega\left(n+1\right)$, where $n = n_x + n_y$, and degeneracy $g_n = n+1$.

(c) Define creation operators $\hat{a}_\pm^\dagger \equiv (\hat{a}_x^\dagger \pm i\hat{a}_y^\dagger)/\sqrt{2}$. Show that the z-component of orbital angular momentum of the particle in the well is $\hat{L}_z = \hbar\left(\hat{a}_+^\dagger \hat{a}_+ - \hat{a}_-^\dagger \hat{a}_-\right)$. Interpret the physical meaning of $\hat{a}_\pm^\dagger \equiv (\hat{a}_x^\dagger \pm i\hat{a}_y^\dagger)/\sqrt{2}$.

(d) Show that $\left|n_+\right\rangle \otimes \left|n_-\right\rangle = \frac{\left(\hat{a}_+^\dagger\right)^{n_+} \left(\hat{a}_-^\dagger\right)^{n_-}}{\sqrt{n_+! n_-!}} \left|0_+\right\rangle \otimes \left|0_-\right\rangle$ is an energy eigenvector, with eigenvalue $E_n = \hbar\omega\left(n+1\right)$, where $n = n_+ + n_-$, and eigenvector of $\hat{L}_z$ with eigenvalue $m\hbar = (n_+ - n_-)\hbar$.

(e) Thus define joint eigenstates of mutually commuting operators $(\hat{N}, \hat{L}_z)$, $\left|n,m\right\rangle$. Show that $\left|n,m\right\rangle = \left|n_+ = \frac{n+m}{2}\right\rangle \otimes \left|n_- = \frac{n-m}{2}\right\rangle$, and from this determine the possible values *m*, and thus the degeneracy of the state.