Physics 522, Spring 2016
Problem Set #4
Due: Thursday Feb. 18, 2016 @ 5PM

Problem 1: Cohen-Tannoudji et al. Vol. I, Prob. 5, p. 767 (10 Points).

5. A system whose state space is $\mathscr{E}_{\mathbf{r}}$ has for its wave function:

$$\psi(x, y, z) = N(x + y + z)\, e^{-r^2/\alpha^2}$$

where α, which is real, is given and N is a normalization constant.

a. The observables L_z and $\mathbf{L}^2$ are measured; what are the probabilities of finding 0 and $2\hbar^2$? Recall that:

$$Y_1^0(\theta, \varphi) = \sqrt{\frac{3}{4\pi}} \cos\theta$$

b. If one also uses the fact that:

$$Y_1^{\pm 1}(\theta, \varphi) = \mp \sqrt{\frac{3}{8\pi}} \sin\theta\, e^{\pm i\varphi}$$

is it possible to predict directly the probabilities of all possible results of measurements of $\mathbf{L}^2$ and L_z in the system of wave function $\psi(x, y, z)$?

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Problem 2: Cohen-Tannoudji et al. Vol. I, Prob. 6, p. 768 (20 Points).

6. Consider a system of angular momentum $l = 1$. A basis of its state space is formed by the three eigenvectors of L_z: $|+1\rangle$, $|0\rangle$, $|-1\rangle$, whose eigenvalues are, respectively, $+\hbar$, 0, and $-\hbar$, and which satisfy:

$$L_\pm |m\rangle = \hbar\sqrt{2}\, |m \pm 1\rangle$$
$$L_+ |1\rangle = L_- |-1\rangle = 0$$

This system, which possesses an electric quadrupole moment, is placed in an electric field gradient, so that its Hamiltonian can be written:

$$H = \frac{\omega_0}{\hbar}(L_u^2 - L_v^2)$$

where L_u and L_v are the components of $\mathbf{L}$ along the two directions Ou and Ov of the xOz plane which form angles of 45° with Ox and Oz; ω_0 is a real constant.

a. Write the matrix which represents H in the $\{ |+1\rangle, |0\rangle, |-1\rangle \}$ basis. What are the stationary states of the system, and what are their energies? (These states are to be written $|E_1\rangle$, $|E_2\rangle$, $|E_3\rangle$, in order of decreasing energies.)

b. At time $t = 0$, the system is in the state :

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}[|+1\rangle - |-1\rangle]$$

What is the state vector $|\psi(t)\rangle$ at time t? At t, L_z is measured; what are the probabilities of the various possible results?

c. Calculate the mean values $\langle L_x \rangle(t)$, $\langle L_y \rangle(t)$ and $\langle L_z \rangle(t)$ at t. What is the motion performed by the vector $\langle \mathbf{L} \rangle$?

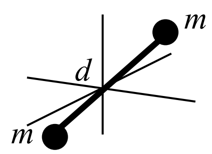
d. At t, a measurement of L_z^2 is performed.

(*i*) Do times exist when only one result is possible?

(*ii*) Assume that this measurement has yielded the result $\hbar^2$. What is the state of the system immediately after the measurement? Indicate, without calculation, its subsequent evolution.

Problem 3: The rigid rotator (15 points)

Consider a dumbbell model of a diatomic molecule, with two masses attached rigidly to a massless rod of length *d*.



(a) Assuming *d* cannot change, and its center of mass does not change, show that the Hamiltonian is

$$\hat{H} = \frac{\hat{L}^2}{2I^2}$$

where $\hat{L}^2$ is the squared angular momentum and *I* is the moment of inertia of the masses for rotation perpendicular to the dumbbell.

(b) What are the energy levels of the system? What is their degeneracy? What are the energy eigenfunctions? Sketch a level diagram.

(c) Diatomic nitrogen, with d=100 pm. Suppose a quantum jump occurs from level denoted by quantum number l+1 to l. What this the wavelength of the emitted?

(d) Suppose you measured the spectrum of emitted light between different transitions. Explain how you would you it to measure the moment of inertia of the molecule.