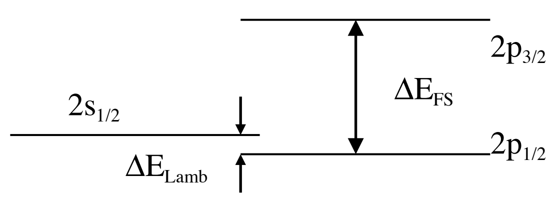
Physics 522, Spring 2016
Problem Set #8
Due: Thursday Mar. 31, 2016 @ 5PM

Problem 1: Addition of spin and orbital angular momentum (15 Points)
Consider an electron with orbital angular momentum quantum number $l = 1$ and spin quantum number $s = 1/2$.
(a) Using the brute force diagonalization method discussed in Lecture find the simultaneous eigenvectors of j^2, $\hat{j}_z$, $\hat{s}^2$, $\hat{l}^2$.

(b) Check your answer by using the Clebsch-Gordan coefficients.

(c) Explicitly calculate the matrix elements of $\hat{\vec{l}} \cdot \hat{\vec{s}}$ in the coupled basis. Show that this what you expect based on $\hat{j}^2 = \hat{l}^2 + \hat{s}^2 + 2\hat{\vec{l}} \cdot \hat{\vec{s}}$.

Problem 2: Stark Shift in Hydrogen (20 points)
Excluding nuclear spin, the *n*=2 manifold in Hydrogen has the configuration:



where $\Delta E_{FS}/h$=10 GHz (the fine structure splitting) and $\Delta E_{Lamb}/h$=1 GHz (the Lamb shift – an effect of quantum fluctuations of the electromagnetic field). In class we neglected these shifts when calculating the Stark shift. This was valid if $ea_0 E_z >> \Delta E$.
Let $x \equiv ea_0 E_z$.

(a) Suppose $x \tilde{<} \Delta E_{Lamb}$, but $x << \Delta E_{FS}$. Then we need only consider the ($2s_{1/2}$, $2p_{1/2}$) subspace in a near degenerate case. Find the new energy eigenvectors and eigenvalues to first order. Are they degenerate? For what value of the field (in volts/cm) is the level separation doubled over the zero field Lamb shift?
(*Hint:* Use the representation of the fine structure eigenstates in the uncoupled representation)

(b) Now suppose $x \tilde{>} \Delta E_{FS}$. We must include all states ($2s_{1/2}$, $2p_{1/2}$, $2p_{3/2}$) in the near degenerate case. Calculate and plot numerically the eigenvalues as a function of x, in the range from

0 GHz < x < 10 GHz.

(c) Comment on the behavior of these curves. Do they have the expected asymptotic behavior? Find analytically the eigenvectors in the limit $x / \Delta E_{FS} \rightarrow \infty$. Show that these are the expected perturbed states.