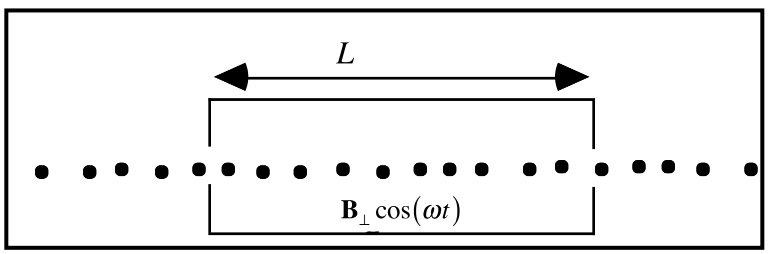
Physics 522, Spring 2016
Problem Set #12
Due: Thursday Apr. 21, 2016 @ 5PM

Problem 1: Magnetic Resonance: Rabi vs. Ramsey (25 Points)

The technique of measuring transition frequencies with magnetic resonance was pioneered by I. I. Rabi in the late 30's. It was modified by Ramsey (his student) about 10 years later, and now serves as the basis for atomic clocks and the SI definition of the second. All precision atomic measurements, including modern atom-interferometers and quantum logic gates in atomic systems, have at their heart a Ramsey-type geometry.

(i) Rabi resonance geometry. Consider a beam of two-level "spins" with energy splitting $\hbar\omega_0$ passing through an "interaction zone" of length L, in which they interact with a monochromatic field oscillating at frequency ω that drives transitions between $|\downarrow\rangle$ and $|\uparrow\rangle$.

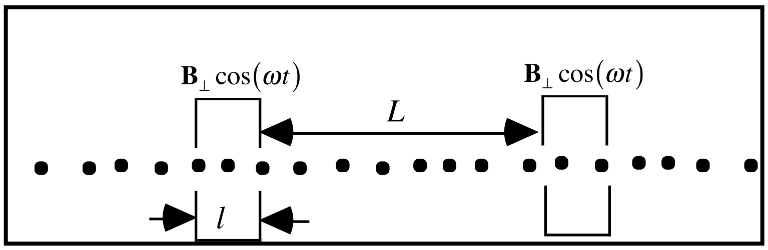


(a) Suppose all the spins start in the state $|\downarrow\rangle$, and have a well defined velocity v, chosen such that $\Omega L / v = \pi$, where $\Omega = \gamma B_\perp / 2$ is the now the bare Rabi frequency. Plot the probability to be in the excited state $|\uparrow\rangle$, $P_\uparrow$ as a function of driving frequency ω. What is the "linewidth" of $P_\uparrow$? Explain your plot in terms of the Bloch-sphere.

(b) Now suppose the spins have a distribution of velocities characteristic of thermal beams: $f(v) = \frac{2}{v_0^4} v^3 \exp(-v^2 / v_0^2)$, where $v_0 = \sqrt{2k_B T / m}$. For Δ=0, plot $P_\uparrow$ vs. L / L_0 where $L_0 = v_0 / \Omega$ (you may need to do this numerically). At what L is it maximized - explain? Also plot as in (a), $P_\uparrow$ as a function of ω with $L = L_{\max}$. What is the linewidth? Explain in terms of the Bloch-sphere.

(ii) Ramsey separated zone method

As you have seen in parts (a)-(b), assuming one can make the velocity spread sufficiently small, the resonance linewidth is limited by the interaction time L/v. This is known as "transit-time broadening" and is a statement of the time-energy uncertainty principle. Unfortunately, if we make L larger and larger other inhomogeneities, such as the amplitude of the driving field come into play. Ramsey's insight was that one can in fact "break up" the π-pulse given to the atoms into two π/2-pulses in a time $\tau=l/v$ (i.e. $\Omega\tau = \pi/2$), separated by *no interaction* for a time $T=L/v$. The free interaction time can then made *much* longer.



(c) Given a mono-energetic spins with velocity v, internal state $|\psi(0)\rangle = |\downarrow_z\rangle$, and field at a detuning $|\Delta| << \Omega$ so that $\Omega_{tot} = \sqrt{\Omega^2+\Delta^2} \approx \Omega$ find:

$$|\psi(\tau = l/v)\rangle,\ |\psi(\tau + T = (l+L)/v)\rangle,\ |\psi(2\tau + T = (2l+L)/v)\rangle$$

and show that mapping of the state on the Bloch-sphere.

(d) Plot $P_\uparrow(t_{final} = 2\tau + T)$ as a function of ω. Plot also for the case of finite spread in velocity as in part (b). What is the linewidth?

Problem 2: Adiabatic Passage (10 Points)

(a) Qualitatively suppose we apply a radiation field with a frequency well below resonance ($\Delta <<0$) and sweep the field slowly up through resonance, ending well above resonance ($\Delta >> 0$), on a time scale much slower that the Rabi frequency $T>>\Omega^{-1}$, but fast compared to spontaneous emission $T<<\Gamma^{-1}$. Use the Bloch-sphere magnetic resonance picture to show that population in the ground state will "adiabatically" be transferred to the excited state.

(b) Quantitatively, sketch the eigenvalues of the two level atom in the RWA as a function of the frequency of the laser. Use the adiabatic theorem of quantum mechanics to explain the transfer of population from ground to excited, and the constraints on the time scales.