Physics 522, Spring 2016

Problem Set #13: EXTRA CREDIT

Due: Tuesday May 10, 2016 @ 5PM

Problem 1: Photon absorption cross-section and photon scattering (20 points)
Given a laser beam at frequency ω_L incident of an atom, we can define the absorption cross-section in the usual way,

$$\sigma_{abs}(\omega_L) = \frac{P_{abs}}{I_{inc}},$$

where I_{inc} is the incident intensity and P_{abs} is the absorbed power Assume a "weak" incident (polarized) monochromatic (narrow band) plane wave near resonance to a two-level $|g = nS\rangle \to |e = nP\rangle$ transition for a time T>>1/Γ, where Γ is the atomic linewidth.

(a) Use Fermi's Golden rule to show that

$$\sigma_{abs}(\omega_L) = \frac{4\pi^2}{\hbar c}\left|\langle e|\hat{\mathbf{d}}\cdot\vec{\varepsilon}_L|g\rangle\right|^2 \omega_L\, g(\omega_L)$$,where $g(\omega_L)$ is the normalized atomic

lineshape, and for the case of lifetime broadening, $\sigma_{abs}(\omega_L) = \dfrac{\sigma_0}{1+4\Delta^2/\Gamma^2}$, where

$$\sigma_0 = \frac{8\pi\omega_L}{\hbar c}\frac{\left|\langle e|\hat{\mathbf{d}}\cdot\vec{\varepsilon}_L|g\rangle\right|^2}{\Gamma}$$ is the absorption cross-section on resonance (Δ=0).

(b) Using the expression for the spontaneous emission rate (Einstein-A coefficient), $\Gamma = \dfrac{4}{3\hbar}k_L^3\left|\langle e|\mathbf{d}|g\rangle\right|^2$, show that the resonant absorption cross section is,

- $\sigma_0 = 3\lambda^2/2\pi$, when the field is polarized (ignore spin of the atom)
- $\sigma_0 = \lambda^2/2\pi$ if the field and/or the atom are *unpolarized*.

The bottom line here is that the resonant cross section is of the order of the square of the exciting wave length, **much** larger that the physical cross section of the atom itself for optical resonance.

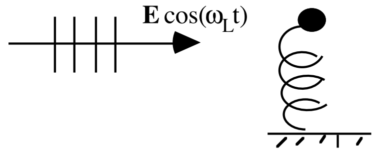
(c) If a photon is absorbed by an atom is will eventually be emitted. This is not necessarily true of other matter, e.g. a molecule or condensed mater (liquid or solid) where the excited matter can relax with other degrees of freedom such as vibrational motion (phonons) etc. The absorption of a photon followed by spontaneous emission is a random direction is *photon scattering*.

Argue that for weak fields, the scattering cross-section is equal to the absorption cross-section, $\sigma_{scatt}(\omega) = \sigma_{abs}(\omega)$, and show that the *scattering-rate* is,

$$\gamma_{scatt} = \frac{I}{\hbar\omega}\sigma_{scatt} = \frac{s}{2}\Gamma,$$ where $s = \dfrac{\Omega^2/2}{\Delta^2+\Gamma^2/4}$ is the saturation parameter.

Problem 2: Classical Lorentz oscillator model of photon scattering (20 points)

Consider the scattering of an electromagnetic wave by a damped Lorentz oscillator



(a) The absorption cross section, σ_{abs}, is defined as the rate at which energy is absorbed by an atom, divided by the incident flux of energy, the intensity $I = \frac{c}{8\pi}|\mathbf{E}_0|^2$ (CGS units). Show that the classical model of absorption gives,

$$\sigma_{abs,class} = \frac{2\pi^2 e^2}{mc} g(\omega_L), \text{ where } g(\omega) = \frac{\Gamma_{rad}/(2\pi)}{(\omega-\omega_{eg})^2 + \Gamma_{rad}^2/4} \text{ is the line shape.}$$

Assume near resonance so that $\Delta = \omega_L - \omega_0 << \omega_L, \omega_0$.

(b) In the case of radiative damping, all energy absorbed is re-radiated, and is thus *scattered*. Use standard scattering theory to derive the differential scattering cross section for the Lorentz oscillator model, $\frac{d\sigma_{scat}}{d\Omega}$, and after integrating over all solid angles, show that the total scattering cross section equals the absorption cross section found in part (a). Here take $\Gamma_{rad} = \frac{2}{3}\frac{e^2}{mc^3}\omega_0^2$.

(c) We can re-derive the expression for the classical natural linewidth Γ_{rad} that we found in class via radiation reaction by looking directly at energy conservation in the scattering process. For the field on resonance, equate the time averaged absorbed power (rate at which field does work on electron, averaged over a period of oscillation) to the Larmor formula for the averaged radiated power to show,

$$\Gamma_{rad} = \frac{2}{3}\frac{e^2}{mc^3}\omega_0{}^2 = \frac{2}{3}\left(k_0 r_c\right)\omega_0, \text{ where } r_c \text{ is the classical electron radius.}$$

Evaluate this for the case the sodium "D2 resonance" (the yellow light in stree light), of excitation wavelength is 589 nm. The quantum decay rate is $\Gamma/2\pi = 9.8$ MHz. What is the oscillator strength of the transition?

(d) Show that the scattering cross section can be rëexpressed as $\sigma_{scat} = \frac{6\pi \lambda\!\!\!^{-}{}_0^2}{1+\left(4\Delta^2/\Gamma_{rad}^2\right)}$, where $\lambda\!\!\!^{-}{}_0 = \lambda/2\pi$. This expression holds true quantum mechanically as well with $\Gamma_{rad} \to \Gamma$.