I. Deutsch

COURSE 2

LASER COOLING, OPTICAL TRAPS AND OPTICAL MOLASSES

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Introduction

This course is intended as a complement to the course in laser cooling and trapping given at this school by Claude Cohen-Tannoudji [1] as well as to those of Herbert Walther [2] and of Rainer Blatt [3]. Here I take an "experimental" point of view, discussing how specific atom manipulation experiments work and how well experiments agree with the theory. The focus of this course is on those techniques of cooling and trapping that are appropriate for application to neutral atoms. Approaches used for ions are covered in refs. [2] and [3].

The course is divided into three main parts. In the first I treat "Doppler cooling" as a result of the velocity dependence of the radiation pressure force and show the common features of a number of experiments which reduce the thermal velocity spread of a group of atoms. The second part deals with the position-dependence of radiative forces, and with the principles of various kinds of laser traps. The third part summarizes the experiments on optical molasses with particular emphasis on comparing these results with the predictions concerning the recently identified polarization gradient cooling mechanisms.

1. Doppler cooling

1.1. The Doppler shift

Laser cooling is the reduction of atomic velocity spread through the action of radiative forces. Such a reduction is necessarily achieved by a velocity dependence of the force. In Doppler cooling this dependence comes about because the Doppler shift experienced by a moving atom changes the detuning of that atom's resonant frequency from the frequency of the applied laser light. This in turn changes the photon absorption rate and hence the force on the atom.

While the Doppler shift is often derived as the result of a transformation between relatively moving frames, it can also be understood as the result of conservation of energy and momentum. Take the non-relativistic limit

and consider the one-dimensional (1-D) example of an atom of mass M moving with an initial velocity v_i and an impinging photon of frequency ω_L and wavevector $k_L = \omega_L/c$. The internal energy difference between the atomic ground and excited states is $\hbar\omega'_A$. For resonant absorption of the photon we equate momentum and energy before and after absorption: $Mv_i + \hbar k = Mv_f$ and $Mv_i^2/2 + \hbar\omega_L = Mv_f^2/2 + \hbar\omega'_A$. The velocity change $v_f - v_i$ is the recoil velocity $v_R = \hbar k/M$. The required frequency shift $\omega_L - \omega'_A = kv_i + \hbar k^2/2M$. The first term is the familiar Doppler shift $kv = \omega_L v/c$, and the second term is called the recoil shift, being equal to the kinetic energy, in frequency units, imparted to an atom initially at rest when it absorbs a photon. For most of what follows in the course, the recoil energy E_R will be small, and in any case we will absorb it into the definition of the atomic resonant frequency: $\omega_A = \omega'_A + \hbar k^2/2M$. The preceding discussion is easily generalized in 3 dimensions to give the usual nonrelativistic Doppler shift $\delta_{Dop} = \boldsymbol{k}\cdot\boldsymbol{v}$. This is the amount by which the laser frequency must be increased to be resonant with an atom at velocity v.

1.2. Radiation pressure

For a laser frequency already detuned from the zero velocity resonance by $\delta = \omega_A - \omega_L$ the effective detuning is $\delta_{eff} = \delta - \boldsymbol{k}\cdot\boldsymbol{v}$. A two-level atom irradiated by a plane wave laser beam having this detuning experiences an average force in the direction of propagation:

$$F = \hbar k \frac{\Gamma}{2} \frac{I/I_0}{1 + I/I_0 + \left[\frac{2(\delta - \boldsymbol{k}\cdot\boldsymbol{v})}{\Gamma}\right]^2}. \tag{1}$$

Here Γ is the natural decay rate of the excited state population and is the natural full width at half maximum (FWHM) of the resonance. I is the laser intensity and I_0 is the saturation intensity, although some authors refer to $2I_0$ as the saturation intensity. $I/I_0 = 2\Omega_1^2/\Gamma^2$, the normalized intensity, where Ω_1 is the Rabi frequency. Also, the reader is cautioned that some authors refer to I/I_0 or $I/2I_0$ as the "saturation parameter." Here and in ref. [1] saturation parameter means the detuning-dependent quantity (see eq. (3.6) of ref. [1])

$$s = \frac{I/I_0}{1 + (2\delta_{eff}/\Gamma)^2}. \tag{2}$$

The fractional population in the excited state is then (eq. (3.7) of ref. [1])

$$f = \frac{1}{2}\frac{s}{1+s}. \tag{3}$$

Equation (1) is simply a re-writing of eq. (3.25) derived in ref. [1]. It is valid for a two-level atom in an arbitrarily intense plane wave. It expresses the force as the product of the photon momentum $\hbar k$ and the photon scattering rate, that is, the rate of events involving a photon absorption followed by a spontaneous emission. These transfer an average of one laser photon momentum to the atom since the spontaneous emission is symmetrically distributed. Absorption followed by stimulated emission does not contribute to the force in a plane wave because the stimulated emission is in the same direction as the laser propagation. Because of this, the force (1) is often called the spontaneous force. It is also referred to as the radiation pressure force or scattering force. It is the velocity dependence of this force which leads to Doppler cooling. At high intensity this force saturates to the value $\hbar k\Gamma/2$. The spontaneous force is limited by the rate at which spontaneous emissions can occur. These occur at a rate Γ for excited atoms whose maximum fractional population is 1/2 according to eq. (3).

The acceleration of an atom due to the saturated radiation pressure force is $a_{max} = \hbar k\Gamma/2M = v_R\Gamma/2$, which can be quite large. For sodium with $\lambda = 2\pi/k = 589$ nm, $1/\Gamma = 16$ ns, and $M = 23$ amu, $v_R \approx 3\,\mathrm{cm/s}$ and $a_{max} \approx 10^6\,\mathrm{m/s^2}$. For hydrogen, with $\lambda = 122\,\mathrm{nm}$, $1/\Gamma = 1.6\,\mathrm{ns}$, and $M = 1\,\mathrm{amu}$, $v_{rec} \approx 3\,\mathrm{m/s}$ and $a_{max} \approx 10^9\ \mathrm{m/s^2}$. This acceleration would stop a thermal, 1000 m/s Na atom in 1 ms over 0.5 m, and a thermal, 2.5×10^3 m/s H atom in 2.5 μs over 3 mm.

1.3. Deflection of an atomic beam

Some of the most important features of Doppler cooling can be illustrated in the problem of deflecting an atomic beam. Consider, as in fig. 1a, an atomic beam of initial velocity $\boldsymbol{v}_i$ entering a perpendicularly directed laser beam tuned to resonance ($\delta = 0$). The exact solution to the atomic motion is complicated, but we can see that, according to eq. (1) the initial acceleration is

$$\dot{v}_i = \frac{\hbar k\Gamma}{2M}\frac{I/I_0}{1 + I/I_0}. \tag{4}$$

As the velocity changes, the acceleration will remain of this order as long as $\boldsymbol{k}\cdot\boldsymbol{v} \leqslant \Gamma\sqrt{1 + I/I_0}$. As $\boldsymbol{k}\cdot\boldsymbol{v}$ becomes larger than this the force will

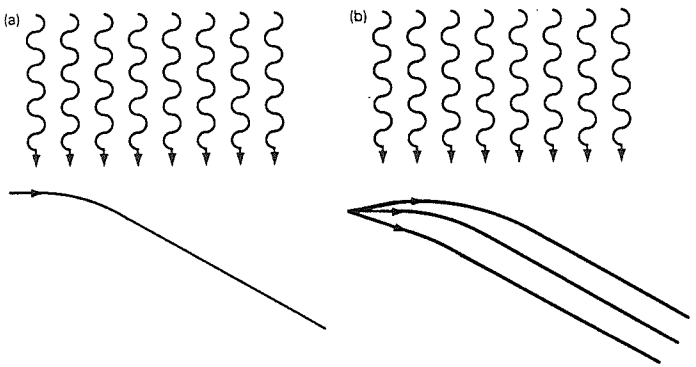

Fig. 1. (a) Laser deflection of an atomic beam. (b) Cooling a diverging beam.

decrease rapidly because the atom will be out of resonance. If $I/I_0 \simeq 1$, the time required to accelerate the atom out of resonance is of the order of $\hbar/E_R$ where E_R is the recoil energy. This is exactly the "external" time scale discussed in ref. [1]. It is about 6 μs for sodium and 11 ns for hydrogen.

Because the atom goes out of resonance the velocity can be changed by only a few times Γ/k unless the intensity is very high. This is 6 m/s for Na and 12 m/s for H. The limitation on the velocity change is one of the biggest problems in manipulating atoms with lasers, but it leads to one of the biggest advantages: Doppler cooling.

Figure 1b shows an atomic beam with a spread of transverse velocities encountering a transversely directed laser beam. For initial velocities toward the laser ($\boldsymbol{k}\cdot\boldsymbol{v} < 0$) the atom is out of resonance on the blue ($\delta_{\text{eff}} > 0$) side, and is accelerated into resonance ($\boldsymbol{k}\cdot\boldsymbol{v} = 0$) and then out of resonance on the red side ($\boldsymbol{k}\cdot\boldsymbol{v} > 0$). Atoms with initial $\boldsymbol{k}\cdot\boldsymbol{v} = 0$ are accelerated out of resonance, and atoms initially with $\boldsymbol{k}\cdot\boldsymbol{v} > 0$ are already out of resonance on the red side and are accelerated only a small amount further out of resonance. The net effect is that all atoms emerge with nearly the same transverse velocity, the one needed to put them just out of resonance on the red side. This compression of velocities is a transverse Doppler cooling.

This transverse cooling (usually called "collimation") is generally welcome, but the deflection, or change in transverse velocity is still limited. To overcome the limitation, we need to compensate for the change in Doppler shift as the laser acts on the atom. One way of accomplishing this compensation is shown in fig. 2. Here, the atomic beam encounters not a plane wave but a converging laser beam (focused by a cylindrical lens). For a proper choice of parameters the beam will be deflected in such a way that

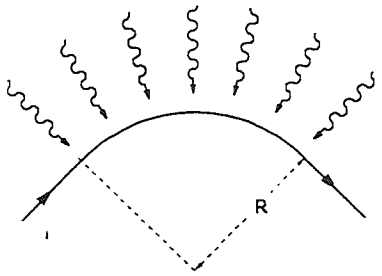

Fig. 2. Laser deflection with a converging laser beam.

it always remains perpendicular to the local direction of propagation of the light. In this way the changing direction of the velocity is compensated by the changing direction of $\boldsymbol{k}$ and the Doppler shift does not change. This procedure has been used [4] to produce deflections as large as 30° in a Na beam with an incident velocity of 100 m/s at $R = 62$ mm.

To satisfy this condition, the radiation pressure force, given by eq. (1) with $\boldsymbol{k}\cdot\boldsymbol{v} = 0$ must equal the required centripetal force Mv^2/R. If this condition is not exactly fulfilled, the deflection process can self-correct so as to fulfill it. This can be seen qualitatively from fig. 2. If the atomic velocity entering the laser beam is too large to be deflected along the indicated path, the atoms will go to a larger radius while being decelerated so that the condition is satisfied at a larger R and a smaller tangential velocity. Let us now consider an atom with a tangential velocity v_θ that satisfies the condition for circular deflection at a radius R, but which has a small radial velocity v_r. We also assume that R is large enough that we can neglect the effect of small changes in R. Now we transform to a reference frame rotating at $\omega = v_\theta/R$. In this frame the tangential velocity is zero and the total radial force on the atom is

$$F = \frac{Mv_\theta^2}{R} - \hbar k \frac{\Gamma}{2}\frac{I/I_0}{1 + I/I_0 + (2\delta/\Gamma)^2} + \frac{\partial F}{\partial v_r} v_r. \tag{5}$$

The first term is the centrifugal force in the accelerated frame, which by assumption cancels the second term, the radiation pressure force for zero v_r. The third term is the first order correction to the radiation pressure for non-zero v_r. By differentiating eq. (1), we determine this term to be

$$F_r = 2\hbar k^2 \frac{I}{I_0} v_r \frac{2\delta/\Gamma}{[1 + I/I_0 + (2\delta/\Gamma)^2]^2}. \tag{6}$$

If $\delta < 0$ this is a force that is proportional to and opposes v_r. It acts like a viscous friction force and will damp small radial velocities to zero, leading to a stable orbit. If an atomic beam with a spread of radial velocities enters the configuration of fig. 2, that spread will be reduced by the friction force in the rotating frame. This is again transverse Doppler cooling as in fig. 1b. There the $\delta < 0$ condition was insured by the fact that atoms were pushed out of resonance with the laser. Here, in the rotating frame, the centrifugal force compensates the radiation pressure force, the atoms stay near resonance, and only the action of cooling or the friction force is evident. Experiments using this geometry to produce both large deflections and significant cooling are described in refs. [4] and [5].

A complete treatment of this kind of experiment for a spread of incident velocities would be complicated because we need to consider different values of R, the variation of I/I_0 with R and the fact that there is no unique transformation to make the tangential velocity zero. Let us consider a different, 1-D problem: laser deceleration and cooling of an atomic beam.

1.4. Deceleration of an atomic beam

Consider the deceleration of a collimated atomic beam by a counterpropagating plane wave laser beam. The atomic velocities are distributed around a nominal velocity V', with v denoting the difference from V'. The detuning δ' of the laser is chosen so as to be near resonant for atoms at V': $\omega_L - \omega_A = \delta' = -kV' + \delta$, where δ is small. According to eq. (1), the force is

$$F(v) = \hbar k \frac{\Gamma}{2} \frac{I/I_0}{1 + I/I_0 + \left[\frac{2(\delta + kv)}{\Gamma}\right]^2}, \tag{7}$$

and the instantaneous acceleration of an atom having velocity V' is $F(0)/M = a$. For large I/I_0 and $\delta = 0$ the acceleration has a maximum magnitude of

$$a_{max} = \hbar k \frac{\Gamma}{2M}. \tag{8}$$

For sodium atoms, as we saw in the previous section, this is about 10^6 m/s^2, or 10^5 times the acceleration of gravity.

With I/I_0 near unity and $\delta = 0$ an atom at velocity V' will decelerate out of resonance with a characteristic time on the order of T_{ext}. Other atoms

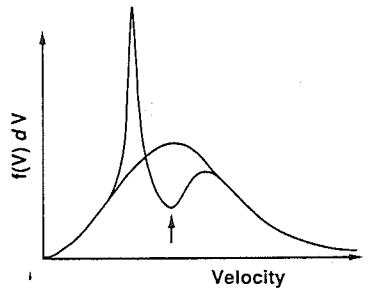

Fig. 3. Longitudinal velocity distribution of an atomic beam before (thin line) and after (bold line) interacting with a counterpropagating, fixed-frequency laser. The arrow indicates V', the velocity resonant with the laser.

with nearby velocities will also decelerate, those with larger velocities first decelerating into resonance, then to still slower velocities out of resonance while initially slower atoms decelerate to still lower velocities. The atoms will "pile up" at a velocity somewhat lower than V'. This situation is analogous to the deflection described in connection with fig. 1b. Both deceleration and cooling occur because a range of velocities around V' is compressed into a narrower range at lower velocity. The change in the velocity distribution of an atomic beam with a thermal spread of velocities is illustrated in fig. 3.

The difficulty with the velocity distribution of fig. 3 is that only a small portion of the total velocity distribution has been decelerated by only a small amount. There are a number of possible solutions to this problem, some of which have been discussed in ref. [6]. Here we discuss "chirp cooling" in which the frequency of the laser is swept up in frequency, or chirped, in time. Because of the chirp, atoms that have been decelerated by the laser stay in resonance, continue to absorb photons, and continue to decelerate. Furthermore, the chirp brings the laser into resonance with additional atoms, having lower velocities than the original group around V'. In this way, the chirp cooling can decelerate and cool atoms having velocities from V' down.

This process can be understood as being analogous to the deflection depicted in fig. 2. We will transform [7] to a frame decelerating with the atoms whose initial velocity was $V' = V'(t = 0)$, and we will assume these atoms to decelerate at a constant rate such that $V'(t) = V'(0) + at$. The chirp is expressed as $\delta'(t) = -kV'(t) + \delta$. This means that the laser has a constant detuning from resonance with atoms decelerating at a, guaranteeing that a is constant. Of course, $a = F(v = 0)/M$, with F given

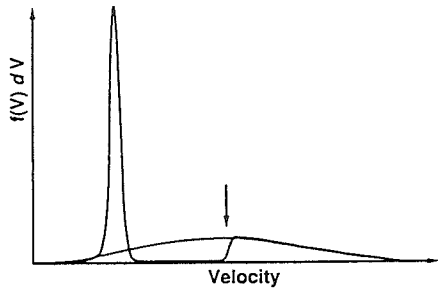

Fig. 4. Longitudinal velocity distribution of an atomic beam before (thin line) and after (bold line) deceleration by a chirped laser. The arrow indicates the velocity initially resonant with the laser.

by eq. (7).

In the decelerating frame the force, in the direction of v', on an atom whose velocity differs by v from the nominal velocity $V'(t)$ is:

$$F(v) = \hbar k \frac{\Gamma}{2} \left[\frac{-I/I_0}{1 + I/I_0 + \left[\frac{2(\delta + kv)}{\Gamma}\right]^2} + \frac{I/I_0}{1 + I/I_0 + \left[\frac{2\delta}{\Gamma}\right]^2} \right]. \tag{9}$$

The second term in the large brackets is the "fictitious" inertial force in the accelerating frame. Expanding this expression for small v, we get:

$$F(v) = 2\hbar k^2 \frac{I}{I_0} \frac{(2\delta/\Gamma)v}{\left[1 + I/I_0 + \left(\frac{2\delta}{\Gamma}\right)^2\right]^2}. \tag{10}$$

The term multiplying v is the friction coefficient $\alpha = M\gamma$. When $\delta < 0$ the force opposes the velocity v and tends to damp all velocities to zero in the decelerating frame, which is $V'(t)$ in the laboratory frame. The final velocity to which the atoms are decelerated is determined in practice by the final frequency to which the laser is chirped. Figure 4 shows the results of chirp cooling an atomic beam. All of the atoms in the initial distribution below the velocity resonant with the laser at the beginning of its chirp are decelerated.

The robust character of this sort of cooling is evident. Atoms within a range of velocities around $V'(t)$ are damped (in velocity) toward V'. Lower velocities, not initially close to V', come within range as the laser chirp brings $V'(t)$ into coincidence with them. If the laser intensity changes during the time an atom is being decelerated (because, for example, the laser beam is not collimated) the atoms will continue to be decelerated according to the chosen chirp rate, but with a different effective detuning δ. That chosen chirp rate, however, must be consistent with an achievable deceleration with the given I/I_0 subject to the condition $\delta < 0$. That is, the chirp rate must satisfy

$$\dot{\delta}' = ka = \frac{\hbar k \Gamma}{2M} \frac{I/I_0}{1 + I/I_0 + \left[\frac{2\delta}{\Gamma}\right]^2} \tag{11}$$

for some δ. And, as we shall see below, it is best if δ is not too small. The condition $a \leqslant a_{\max}$ (eq. (8)) implies an upper limit to the allowable chirp rate.

1.5. Optical molasses in one dimension

In the examples of deflection and deceleration given above, we transformed into an accelerating frame where the radiation pressure force was opposed and compensated by an inertial force in that frame. The total force on zero velocity atoms in the chosen frame is zero (eqs. (5, 6) and (9, 10)), and a proper choice of detuning makes this velocity a point of stable equilibrium. If we wish to have the same situation in the laboratory rest frame, we must have a real force to compensate the radiation pressure. This force might be provided by an electric field if the atoms are charged [8], or more generally, by an opposing radiation pressure.

A configuration of counterpropagating laser beams used for cooling of atoms has come to be known as "Optical Molasses" [9]. If the intensity of each beam is small ($I/I_0 \ll 1$) we can write the total force on an atom as the sum of the radiation pressure from each of the two beams, as long as we understand this to be the force averaged over a wavelength of the light. In one dimension:

$$F(v) = \hbar k \frac{\Gamma}{2} \frac{I/I_0}{1 + \left[\frac{2(\delta - kv)}{\Gamma}\right]^2} - \hbar k \frac{\Gamma}{2} \frac{I/I_0}{1 + \left[\frac{2(\delta + kv)}{\Gamma}\right]^2}. \tag{12}$$

Figure 5 shows the force from each of the beams, and the total force for a variety of detunings.

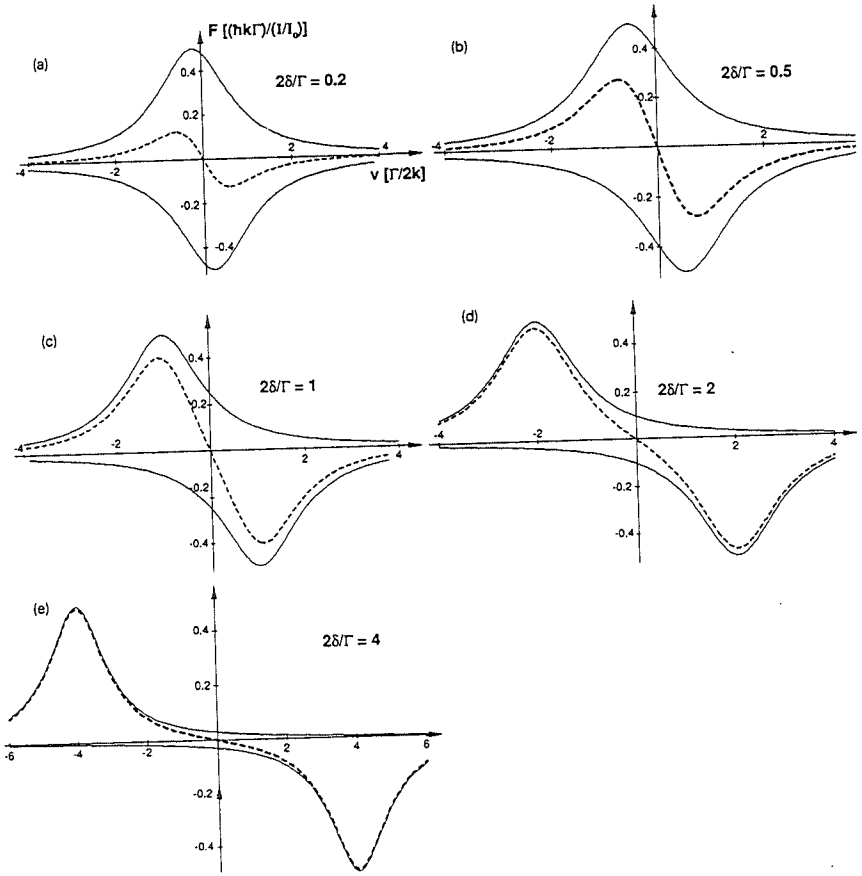

Fig. 5. Weak standing wave radiation pressure force (in units of $\frac{(\hbar k\Gamma)}{(I/I_0)}$) versus velocity (in units of $\Gamma/2k$) for various values of the detuning.

This series of plots illustrates the fact that the slope of $F(v)$ at $v = 0$ has a maximum near $2\delta/\Gamma = 1$. Furthermore, $F(v)$ is linear only over a range of velocities on the order of Γ/k. For small v and I/I_0 the force is given by

$$F(v) = 2\hbar k^2 \frac{(2I/I_0)(2\delta/\Gamma)v}{\left[1 + \left(\frac{2\delta}{\Gamma}\right)^2\right]^2}. \tag{13}$$

The term multiplying v in eq. (13) is the friction coefficient $\alpha = \gamma M$. The characteristic time for damping the atomic velocity is γ^{-1}. For a given, small value of I/I_0 the damping time is minimized for $2\delta/\Gamma = 1/\sqrt{3}$. With this detuning, $\tau_{\rm damp} = \gamma^{-1} \approx T_{\rm ext}(I_0/I)$, where $T_{\rm ext} = \hbar/E_{\rm R}$ is the external time scale. Note that if the saturation parameter

$$s = \frac{I/I_0}{1 + (\frac{2\delta}{\Gamma})^2} \tag{14}$$

is held constant (and small), the friction and the damping rate are maximized at $2\delta/\Gamma = 1$.

For $I/I_0 \ll 1$, eq. (10), derived for cooling with a single travelling wave, is identical to eq. (13) except that here I/I_0 has been replaced with $2I/I_0$. This means that all the results discussed in the context of cooling in a weak standing wave apply also to cooling in the decelerating frame appropriate for chirped cooling.

So far in the discussion of radiation pressure forces we have considered only the time averaged value of the force. The force, of course, results from discrete transfers of momentum when the atom absorbs or emits photons. This discreteness means that the force fluctuates about the average value. The fluctuations tend to heat the atom. To visualize this heating, assume that the average force is zero, but the atom is subject to a fluctuating force of zero mean arising from the emissions and absorptions. Each event transfers momentum to the atom, and in this limit of low intensity, each event is uncorrelated with other events and each momentum transfer is in a random direction, at a random time. The randomness is a direct result of the random character of spontaneous emission. As a result, the atomic momentum undergoes a random walk. The mean square of the momentum increases with time, which is to say the kinetic energy increases, and the atom heats. This diffusion of the atomic momentum is treated in detail in Cohen-Tannoudji's lectures [1]. Here we will follow a simple calculation for weak fields.

Let us calculate the rate of heating in a "truly" one-dimensional situation, a two-level atom in a weak standing wave along the x-axis, with the spontaneous photons assumed to be emitted only along this axis. There are two contributions to the heating: the random direction of the spontaneous emissions, and the randomness of the absorption. Each spontaneously emitted photon goes in a direction which is uncorrelated with that of other emitted photons, so these emissions induce a random walk with momentum step size $\hbar k$. After N emissions, according to the usual random walk result, the mean square momentum is $\langle p_x^2 \rangle = N\hbar^2 k^2$. The momentum diffusion coefficient is defined in terms of the rate of increase of p^2, which

in turn depends on the rate of emitting photons:

$$2D_{\rm spont} = \langle \dot{p_x^2} \rangle = 2\hbar^2 k^2 \frac{\Gamma}{2} \frac{I/I_0}{1+(2\delta/\Gamma)^2}, \tag{15}$$

where the first factor of two comes from the two beams of the standing wave, each having intensity I/I_0. The randomness of the absorption process leads to the direction from which any one photon is absorbed being uncorrelated with the others (recall that we assume $kv, kv_{\rm rec} \ll \Gamma$, so that the Doppler shift due to the recoil velocity does not much change the relative absorption probability.) Thus, $D_{\rm abs} = D_{\rm spont} (= D_{\rm vac})$. This is the same (for the same total intensity) as the result for diffusion in a single running wave in the case where I/I_0 or s is small so that the non-Poissonian Q parameter is small. (Note that for the weak standing wave, we have associated the randomness in absorption with the random direction of absorption, while in a travelling wave, the randomness can be thought of as arising from the randomness in the number of photons absorbed per unit time.)

Now we equate the rate of increase of kinetic energy due to the diffusion with the rate of decrease in kinetic energy due to the damping or friction. Recall that the damping force $F = -\alpha v$, so the cooling rate $\langle Fv \rangle$ is $-\alpha \langle v^2 \rangle$. Thus, in steady state,

$$\langle \dot{E}_{\rm heat} \rangle = \frac{\langle \dot{p^2} \rangle}{2M} = \frac{D}{M} = -\langle \dot{E}_{\rm cool} \rangle = \alpha \langle v^2 \rangle, \tag{16}$$

where D is the total diffusion coefficient, $D_{\rm spont} + D_{\rm abs}$. For this 1-D problem we have a single degree of freedom so $M\langle v^2 \rangle/2 = k_{\rm B}T/2$. This temperature is

$$k_{\rm B}T = \frac{D}{\alpha} = \frac{\hbar\Gamma}{4} \frac{1+(\frac{2\delta}{\Gamma})^2}{(\frac{2\delta}{\Gamma})} = \frac{\hbar\Gamma}{4} \left(\frac{\Gamma}{2\delta} + \frac{2\delta}{\Gamma} \right). \tag{17}$$

The temperature minimizes for $\delta = -\Gamma/2$, giving

$$k_{\rm B}T_{\rm Dop} = \frac{\hbar\Gamma}{2}, \tag{18}$$

where $T_{\rm Dop}$ is called the Doppler cooling limit. Note that this limit is different from ones often given for Doppler cooling in one dimension. In some other treatments the spontaneous emission is not assumed to be along the 1-D axis, but distributed in a manner characteristic of an isotropic, dipole, or other radiation pattern. With such an assumption, which would be appropriate for a "real" 1-D situation such as collimation of an atomic beam along one direction, the 1-D cooling limit is smaller than in eq. (18). The Doppler limit for our idealized 1-D case corresponds to 240 μK, a 1-D rms velocity of 30 cm/s for sodium and 125 μK and 9 cm/s for cesium.

Finally, in accordance with the fact that the cooling in a weak standing wave is equivalent to that in a travelling wave of equal total intensity (assuming we go to an accelerated frame, or the radiation pressure is otherwise compensated), the Doppler cooling limit in a weak travelling wave is the same as in a weak standing wave. There has been some confusion on this point because of the way in which $D_{\rm abs}$ is considered. Here we see it as arising from the fluctuations in the absorbed photons. In a standing wave, when one takes account of the variation of intensity along the standing wave, it may also be thought of as a fluctuation of the stimulated or dipole force [10]. (See section 2.6 of these notes and see also Cohen-Tannoudji's lectures.) In the absence of a standing wave, it might appear that there is no dipole force, and thus no fluctuation of that force, so the momentum diffusion and the Doppler temperature should be less. In fact, as we have seen above, this is not so.

In the above treatment, we have consistently taken $I/I_0 \ll 1$ in 1-D. This allowed us to treat the standing wave as the sum of two travelling waves. There are treatments of the standing wave case in 1-D for high intensity and a two-level atom (see, for example, refs. [1] and [10]), and for the $J = 0 \rightarrow J = 1$ case [11]. Unfortunately, these treatments do not easily generalize to the 3-D case we wish to treat later. We do have an exact treatment for a single travelling, plane wave at arbitrary intensity, exemplified by eqs. (1,10). This exact result leads to a model, due to Dalibard [12], for treating a strong standing wave that does generalize to 3-D.

Consider two opposed running waves, alternated in time, with 50% duty factor, as illustrated in fig. 6. The average intensity of each wave is I, while the peak intensity is $2I$. We also assume that the time T for which each wave is on is such that $\gamma^{-1} \gg T \gg \Gamma^{-1}$, recalling that γ and Γ are the cooling rate and the natural decay rate, respectively.

Since there is only one wave on at a time, and the transient time Γ^{-1} is negligible, we can simply add the forces from each wave to obtain the total average force:

$$\langle F \rangle = \hbar k \frac{\Gamma}{2} \left[\frac{1}{2} \frac{2I/I_0}{1+2I/I_0+\left[\frac{2(\delta - k\,v)}{\Gamma}\right]^2} - \frac{1}{2} \frac{2I/I_0}{1+2I/I_0+\left[\frac{2(\delta + k\,v)}{\Gamma}\right]^2} \right]. \tag{19}$$

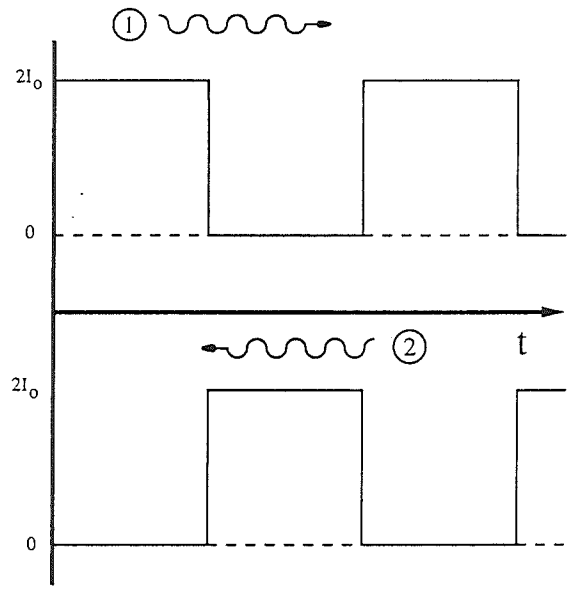

Fig. 6. Counterpropagating, alternated running waves used as a model for a standing wave. The intensities of the beam propagating to the right (1) and left (2) are shown as a function of time.

The factor of 1/2 before each term is from the duty factor, while $2I$ is the instantaneous intensity. Note that the term $2I/I_0$ in the denominators looks as if it is due to saturation and power broadening from the average power summed over both waves, although it is of course due to the instantaneous intensity of a single wave.

As before, we find the friction coefficient:

$$\gamma M = \alpha = 4\hbar k^2 \frac{I}{I_0} \frac{2\delta/\Gamma}{\left[1 + \frac{2I}{I_0} + \left(\frac{2\delta}{\Gamma}\right)^2\right]^2}. \tag{20}$$

This is the same friction coefficient seen in eq. (13), except for the "saturation" factor in the denominator.

We may take this expression for the friction coefficient as exact for the alternated wave case, or as an approximation for the case of continuous counterpropagating waves each of intensity I. The approximation ignores effects due to the standing wave character of the counterpropagating waves such as are treated in, for example, refs. [1,10,12,13]. Nevertheless, it is a fair approximation for $I/I_0 \lesssim 1$. An analysis in ref. [14] compares this model to the exact solutions for a two-level system [1,10] and for the case of counterpropagating waves of opposite circular polarization ($\sigma^+ - \sigma^-$) on a $J = 0 \rightarrow J = 1$ transition [11]. The model agrees moderately well with the $\sigma^+ - \sigma^-$ calculation even for large intensity (and, accidentally, agrees exactly for $I/I_0 = 4$). The model disagrees dramatically with the two-level calculation when $I/I_0 > 1$. This is because the model admits no possibility of absorption from one wave followed by stimulated emission induced by the other. This process does not occur in the $\sigma^+ - \sigma^-$ case, but dominates the behavior of a two-level atom in a standing wave of high intensity.

Now consider the momentum diffusion coefficient in our alternated wave model. In analogy with eq. (15) we can write the spontaneous part of the diffusion as

$$2D_{\text{spont}} = \langle p_x^2 \rangle = \hbar^2 k^2 \frac{\Gamma}{2} \frac{2I/I_0}{1 + 2I/I_0 + (2\delta/\Gamma)^2}. \tag{21}$$

Because of the non-Poissonian nature of the absorption statistics at non-negligible intensity, the part of the diffusion ascribed to the absorption is not the same as that for the spontaneous emission. Assuming as always that the emissions are all along the 1-D axis, we have $D_{\text{abs}} = D_{\text{spont}}(1+Q)$ where Q is Mandel's parameter describing the non-Poissonian character (given as eq. (5.48) in ref. [1]). Thus, the total diffusion $D = D_{\text{spont}}(2+Q)$. For the intensities where the alternating model is a good approximation to a standing wave, $Q \ll 2$, so we will neglect it in the following discussion.

1.6. Optical molasses in N dimensions

This alternated beam model easily generalizes to 2 or 3 dimensions, where we assume a $2N$-fold alternation where N is the dimension (each beam has intensity $2NI$ for $1/2N$ of the time). Then for the friction coefficient along any of the N axes we have

$$\gamma M = \alpha = 4\hbar k^2 \frac{I}{I_0} \frac{2\delta/\Gamma}{\left[1 + \frac{2NI}{I_0} + \left(\frac{2\delta}{\Gamma}\right)^2\right]^2}, \tag{22}$$

and for the total diffusion constant

$$D = 2D_{\text{spont}} = \hbar^2 k^2 \frac{\Gamma}{2} \frac{2NI/I_0}{1 + 2NI/I_0 + (2\delta/\Gamma)^2}. \tag{23}$$

We will use these expressions to obtain approximate results, for moderate intensity, in 3-D optical molasses.

(Note that in eq. (23) the momentum diffusion coefficient has been presented as a scalar. In actuality, it is represented by a tensor $2D_{ij} = \langle p_i p_j \rangle$. For the alternating beam model, there is no correlation between photon scattering from the various beams, so the tensor is diagonal. Furthermore, for a 3-D symmetric situation such as isotropic photon scattering, or dipole scattering induced by three pairs of beams with mutually orthogonal linear polarization, or for our assumption that all spontaneous emissions are along the laser beam axis, the diagonal elements of D are the same in each case, and are all equal, so we may treat D as a scalar.)

For this case where all the axes of the N-D molasses are equivalent, we can write the friction force as $\boldsymbol{F} = -\alpha \boldsymbol{v}$. Then, as in 1-D, $\langle \dot{E}_{\text{heat}} \rangle = D/M$ and $\langle \dot{E}_{\text{cool}} \rangle = -\alpha \langle v^2 \rangle$. From the equipartition theorem we have $N k_{\text{B}} T/2 = M\langle v^2 \rangle/2 = \sum_{i=1}^{N} M\langle v_{\text{i}}^2 \rangle/2$, so the equilibrium temperature is given by

$$k_{\text{B}} T = \frac{D}{N\alpha} = \frac{\hbar\Gamma}{4} \frac{1 + \frac{2NI}{I_0} + (\frac{2\delta}{\Gamma})^2}{(\frac{2\delta}{\Gamma})}. \tag{24}$$

For low intensity this reduces to eq. (17).

We can maximize the friction coefficient given in eq. (22) with respect to both intensity and detuning, finding

$$\gamma M = \alpha = \frac{\hbar k^2}{4N} \text{ for } 2\delta/\Gamma = -1,\ I/I_0 = 1/N. \tag{25}$$

For sodium the minimum damping time is $\gamma^{-1} = 13$ μs in 1-D and 40 μs, in 3-D. For cesium the times are 160 μs and 480 μs respectively. The reduced damping in 3-D is due to the duty factor in the alternated beam model. If we consider this as a model of continuous 3-D molasses, then we may interpret the reduction as being due to the fact that an atom moving along any given axis spends 2/3 of its time interacting with beams perpendicular to its velocity.

With the parameters of eq. (25), eq. (24) gives $k_{\text{B}} T = \hbar\Gamma$, just twice the Doppler cooling limit. In N-D the Doppler limit, achieved at low intensity, is still the lowest temperature obtainable. For either maximum friction or minimum temperature, an atom in N-D has N times as much kinetic energy as in 1-D, but the same energy per degree of freedom.

1.7. Spatial diffusion in optical molasses

We will apply the alternated beam model to study the atomic motion in optical molasses, using a Brownian motion approach. Let us estimate the distance an atom will diffuse in a time t_{d}. Moving at a thermal velocity v_{rms} the atom will travel a distance $l = v_{\text{rms}}/\gamma$ during a damping time. Considering this l as a random walk step which is repeated $t_{\text{d}}\gamma$ times, the mean square distance diffused in t_{d} is

$$\langle r^2 \rangle \approx l^2 t_{\text{d}} \gamma = \frac{D_p t_{\text{d}}}{\alpha^2},$$

where D_p is the momentum diffusion coefficient (the subscript has been added to distinguish it from the spatial diffusion coefficient). More rigorously, we can define a spatial diffusion constant D_x by $\langle x^2 \rangle = 2t_{\text{d}} D_x$. A careful treatment (see refs. [9,14] and references therein) gives $D_x = k_{\text{B}} T/\alpha$ and

$$\langle r^2 \rangle = \frac{2D_p t_{\text{d}}}{\alpha^2}. \tag{26}$$

Using eqs. (22,23) and maximizing the diffusion time for a given diffusion distance we find [14]

$$t_{\text{d}}^{\max} = \frac{4k^2 \langle r^2 \rangle}{27N^2\Gamma} \tag{27}$$

for $2\delta/\Gamma = -1$ and $I/I_0 = 1/(2N)$. The times to diffuse 0.5 cm in 3-D for sodium and cesium are 750 ms and 675 ms, respectively. If we assume the atoms diffuse to the edge of a spherical region (an approximation to the intersection region of three pairs of finite cross section laser beams) and then are lost, we find that the number of atoms within the sphere decays as the sum of exponentials [9,14]. In 3-D the leading term decays with a time constant $t_{\text{M}} = 6t_{\text{d}}/\pi^2$. This is usually referred to as the molasses lifetime.

If an external force is applied to an atom in optical molasses, the atom will acquire a drift velocity such that the friction force cancels the external force:

$$v_{\text{drift}} = F_{\text{ext}}/\alpha. \tag{28}$$

If the external force is gravity we have $v_{\text{drift}} = g/\gamma$. For the maximum damping conditions of eq. (25) in 3-D, the gravity-induced drift is 0.4 mm/s for sodium and 5 mm/s for cesium. This would be quite a significant effect for a centimeter diameter cesium molasses, limiting the molasses lifetime to something on the order of a second. Another source of "external" force might be an imbalance in the intensity of the counterpropagating beams. In ref. [14] an approximate treatment shows that for optimum damping