

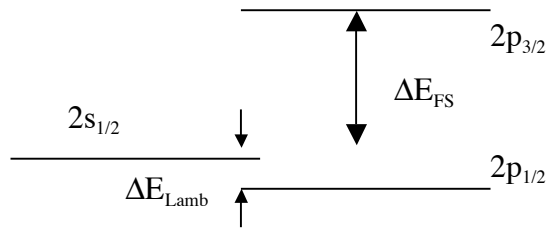
Physics 531

Problem Set #3

Due Tuesday, Feb. 15, 2005

Problem 1: Stark Shift in Hydrogen with Fine Structure

Excluding nuclear spin, the $n=2$ manifold in Hydrogen has the configuration:



where $\Delta E_{FS}/h=10$ GHz (the fine structure splitting) and $\Delta E_{Lamb}/h=1$ GHz (the Lamb shift – an effect of quantum fluctuations of the electromagnetic field). In class we neglected these shifts when calculating the Stark shift. This was valid if $ea_0E_z \gg \Delta E$.

Let $x \equiv ea_0E_z$.

- (a) Suppose $x \lesssim \Delta E_{Lamb}$, but $x \ll \Delta E_{FS}$. Then we need only consider the $(2s_{1/2}, 2p_{1/2})$ subspace in a near degenerate case. Find the new energy eigenvectors and eigenvalues to first order. Are they degenerate? For what value of the field (in volts/cm) is the level separation doubled over the zero field Lamb shift?
(Hint: Use the representation of the fine structure eigenstates in the uncoupled representation which we found in Problem Set #1)
- (b) Now suppose $x \gtrsim \Delta E_{FS}$. We must include all states $(2s_{1/2}, 2p_{1/2}, 2p_{3/2})$ in the near degenerate case. Calculate and plot numerically the eigenvalues as a function of x , in the range from
 $0 \text{ GHz} < x < 10 \text{ GHz}$.

Comment on the behavior of these curves. Do they have the expected asymptotic behavior? Find analytically the eigenvectors in the limit $x / \Delta E_{FS} \rightarrow \infty$. Show these are the expected perturbed states.

Problem 2: Zeeman shift in ground state of hydrogenic atoms

The Zeeman shift is the shift of the energy levels due to an externally applied magnetic field \mathbf{B} . For a one-electron hydrogenic atom, the interaction Hamiltonian is

$$\hat{H}_{\text{int}} = -\vec{\mu}_e \cdot \mathbf{B} - \vec{\mu}_N \cdot \mathbf{B}$$

where $\vec{\mu}_e$, $\vec{\mu}_N$ are the magnetic dipole moments of the electron and nucleus respectively. Let us consider the ground state, where the orbital angular momentum is zero. Then the magnetic moment is due solely to spin angular momentum.

For the case of hydrogen,

$$\vec{\mu}_e = -g_e \mu_B \mathbf{S}, \quad \vec{\mu}_N = -g_p \mu_N \mathbf{I}$$

where \mathbf{S} and \mathbf{I} are the electron and proton spin angular momentum operators (in units of \hbar), μ_B and μ_N are the Bohr and nuclear magnetons, and $g_e = 2$ and $g_p = 5.9$ are the intrinsic “g-factors” for the electron and proton respectively. In this case, because the proton mass is so much larger than that of the electron the nuclear magnetic moment’s interaction with the B-field is negligible compared with the electron’s. Taking \mathbf{B} in the z-direction, the total Hamiltonian, including the magnetic hyperfine interaction is approximately,

$$\hat{H}_{\text{int}} = A \mathbf{I} \cdot \mathbf{S} + 2\mu_B B S_z$$

(a) Diagonalize this Hamiltonian in the 4D ground state manifold. Sketch the energy level diagram as a function of B . Comment on the behavior and the nature of the eigenstates in the limit of large and small B .

(b) Repeat for positronium and muonium. In this case the nuclear mass is *not* much smaller than the lepton and its magnetic moment interaction with the B-field cannot be neglected with respect to the electron’s.

Note, muon g-factor is the approximately same as the electron and the positron’s is equal in magnitude but opposite in sign.