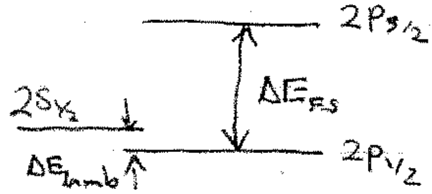
Problem Set #3 Solutions

Problem 1: Stark effect with Fine-structure

$n=2$ manifold of Hydrogen (including fine structure)



Stark effect perturbation: $\hat{H}_{int} = +e\hat{z}E_z$ (quantization axis along $\vec{E}$)

Recall spectroscopic notation: $n l_j \Rightarrow 2s_{1/2}$ ($n=2$, $l=0$, $j=1/2$)

For a given j, there are $2j+1$ degenerate sublevels

$$\begin{cases} 2s_{1/2} \Rightarrow |2s_{1/2}, +1/2\rangle,\ |2s_{1/2}, -1/2\rangle \\ 2p_{1/2} \Rightarrow |2p_{1/2}, +1/2\rangle,\ |2p_{1/2}, -1/2\rangle \\ 2p_{3/2} \Rightarrow |2p_{3/2}, 3/2\rangle,\ |2p_{3/2}, 1/2\rangle,\ |2p_{3/2}, -1/2\rangle,\ |2p_{1/2}, -3/2\rangle \end{cases}$$

Since $\hat{H}_{int}$ acts only on the spatial degree of freedom, it will be useful to reëxpress the eigenstates above in terms of the "uncoupled" angular momentum basis. We did this in P.S. #8, problem 2 (521 Fall 2000). The results were

$$|2s_{1/2}, \pm 1/2\rangle = |2s, \underset{m_l}{0}\rangle \otimes |\underset{m_s}{\pm \tfrac{1}{2}}\rangle$$

$$|2p_{1/2}, \pm 1/2\rangle = \sqrt{\tfrac{1}{3}}\,|2p, 0\rangle \otimes |\pm 1/2\rangle - \sqrt{\tfrac{2}{3}}\,|2p, \pm 1\rangle \otimes |\mp \tfrac{1}{2}\rangle$$

$$|2p_{3/2}, \pm 1/2\rangle = \sqrt{\tfrac{2}{3}}\,|2p, 0\rangle \otimes |\pm 1/2\rangle + \sqrt{\tfrac{1}{3}}\,|2p, \pm 1\rangle \otimes |\mp \tfrac{1}{2}\rangle$$

$$|2p_{3/2}, \pm 3/2\rangle = |2p, \pm 1\rangle \otimes |\pm 1/2\rangle$$

(a) For weak fields $ea_0 E_z \gtrsim \Delta E_{Lamb}$, we can restrict our attention to the $(2s_{1/2}, 2p_{1/2})$ manifold

The matrix representation of $\hat{H}_{int}$ is block diagonal with no off-diagonal ~~elem~~ elements between different m_j as we will see below

Consider $m_j = 1/2$, 2 dim space

$$\hat{H}_0 + \hat{H}_{int} \doteq \begin{array}{c} \begin{bmatrix} \Delta E_L & \epsilon \\ \epsilon^* & 0 \end{bmatrix} \\ |2s_{1/2}\rangle \quad |2p_{1/2}\rangle \end{array}$$

where ΔE_L = Lamb shift

$$\epsilon = \langle 2p_{1/2}, \tfrac{1}{2} | \hat{H}_{int} | 2s_{1/2}, 1/2 \rangle$$

To calculate ϵ, we use the uncoupled representation above:

$$\epsilon = \underset{-eE_z}{\wedge}\sqrt{\tfrac{1}{3}} \langle 2p, 0 | \hat{z} | 2s, 0 \rangle \overset{1}{\cancel{\langle +\tfrac{1}{2} | +\tfrac{1}{2} \rangle}} - \sqrt{\tfrac{2}{3}} \langle 2p, 1 | \hat{z} | 2s, 0 \rangle \overset{0}{\cancel{\langle -\tfrac{1}{2} | +\tfrac{1}{2} \rangle}}$$

orthogonal spin

From class $\langle 2p, 0 | \hat{z} | 2s, 0 \rangle = -3a_0$

$$\Rightarrow \boxed{\epsilon = \mp\sqrt{3}\, e a_0 E_z} \quad \text{(real)}$$

Diagonalize $\hat{H} = \begin{bmatrix} \Delta E_L & 0 \\ 0 & 0 \end{bmatrix} + \epsilon \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$= \frac{\Delta E_L}{2} \hat{\mathbb{1}} + \frac{\Delta E_L}{2} \hat{\sigma}_z + \epsilon\, \hat{\sigma}_x$$

Eigenvalues $\boxed{E_\pm = \frac{\Delta E_L}{2} \pm \sqrt{\frac{(\Delta E_L)^2}{4} + \epsilon^2}}$

$$\boxed{\begin{array}{l}\text{Eigenvectors: } |\pm\rangle = \cos\left(\frac{\Theta}{2}\right)|2p_{1/2}\rangle \pm \sin\left(\frac{\Theta}{2}\right)|2s_{1/2}\rangle \\ \quad \text{where } \tan\Theta = \frac{2\epsilon}{\Delta E_L} \quad (\text{"mixing angle"}) \\ \text{Note: ratio of coupling matrix element to energy separation}\end{array}}$$

New splitting between perturbed $2s_{1/2}$ and $2p_{1/2}$

$$\Delta E_L' = E_+ - E_- = \sqrt{(\Delta E_L)^2 + 4\epsilon^2}$$

Find electric field such that $\Delta E_L' = 2\Delta E_L$

$$\Rightarrow 4\epsilon^2 = 3(\Delta E_L)^2 \Rightarrow \epsilon = \frac{\sqrt{3}}{2}\Delta E_L$$

$$\Rightarrow \sqrt{3}\, e\, a_0 E_z = \frac{\sqrt{3}}{2}\Delta E_L$$

$$\Rightarrow \boxed{E_z = \frac{1}{2ea_0}\Delta E_L}$$

Now for the numbers. Remember, we are using c.g.s. units. The easiest thing to do is express ΔE_L in electron volts, so that $\frac{\Delta E_L}{e}$ is in volts.

Conversion: Planck's constant $h = 4.14 \times 10^{-15}$ eV·s

$$\Rightarrow \Delta E_L = (10^9 \text{ Hz})(4.14\times10^{-15} \text{ eV s}) = 4.14\times10^{-6} \text{ eV}$$

$$a_0 = 0.5\times10^{-8} \text{ cm} \quad (0.5 \text{ Å})$$

$$\boxed{E_z = \frac{4.14\times10^{-6} \text{ V}}{10^{-8} \text{ cm}} = 414 \text{ V/cm}}$$

What about the other m_j substates?

- No off-diagonal matrix elements between different m_j

Proof $\langle 2s_{1/2}\tfrac{1}{2}|\hat{H}_{int}|2p_{1/2},-\tfrac{1}{2}\rangle$

$= +eE_z\left[\left(\langle 2s,0|\langle\tfrac{1}{2}|\right)\hat{z}\left(|2p,0\rangle|-\tfrac{1}{2}\rangle\right) - \sqrt{\tfrac{2}{3}}\left(\langle 2s,0|\langle\tfrac{1}{2}|\right)\hat{z}\left(|2p\,{-1}\rangle|\tfrac{1}{2}\rangle\right)\right]$

$= +eE_z\left[\frac{1}{\sqrt{3}}\langle 2s,0|\hat{z}|2p,0\rangle\underbrace{\langle\tfrac{1}{2}|-\tfrac{1}{2}\rangle}_{0} - \sqrt{\tfrac{2}{3}}\underbrace{\langle 2s,0|\hat{z}|2p,-1\rangle}_{0}\langle\tfrac{1}{2}|\tfrac{1}{2}\rangle\right]$

$= 0$ ✓ and similarly for $\langle 2s\,{-\tfrac{1}{2}}|\hat{H}_{int}|2p_{1/2},\tfrac{1}{2}\rangle$

- The 2×2 matrix representation for $m_j = -\tfrac{1}{2}$ is the <u>same</u> as for $m_j = \tfrac{1}{2}$ (try this yourself).

Thus in the 4-dim subspace of $(2s_{1/2}, 2p_{1/2})$ the representation of $\hat{H}$ is block-diagonal, with two degenerate sub-blocks

$$\hat{H} = \left[\begin{array}{cc|cc} E_L & \epsilon & \multicolumn{2}{c}{\multirow{2}{*}{0}} \\ \epsilon & 0 & & \\ \hline \multicolumn{2}{c|}{\multirow{2}{*}{0}} & E_L & \epsilon \\ & & \epsilon & 0 \end{array}\right] \begin{array}{l} m_j = \tfrac{1}{2} \\ \\ \\ m_j = -\tfrac{1}{2} \end{array}$$

$$\boxed{\text{Thus, the eigenvalue we found above are \underline{doubly} degenerate}}$$

(b) Consider $ea_0 E_z \gtrsim \Delta E_{FS} \Rightarrow$ include all states in $n=2$

Again $\hat{H}$ is block diagonal, with no off-diagonal matrix element between different m_j. These ~~are~~ block are also doubly degenerate for $\pm m_j$. As in class, there are no $p \to p$ matrix elements.

We must thus diagonalize the following 3x3 matrix.

$$\hat{H} \doteq \begin{array}{c} \begin{bmatrix} \Delta E_L & \epsilon & \beta \\ \epsilon & 0 & 0 \\ \beta & 0 & \Delta E_{FS} \end{bmatrix} \\ |2s_{1/2}\rangle \quad |2p_{1/2}\rangle \quad |2p_{3/2}\rangle \end{array} \qquad m_j = \pm 1/2$$

note the $|2p_{3/2}, m_j = \pm 3/2\rangle$ is unperturbed

Here $\beta = \langle 2p_{3/2}, 1/2 | \hat{H}_{int} | 2s_{1/2}, 1/2 \rangle$ (real)

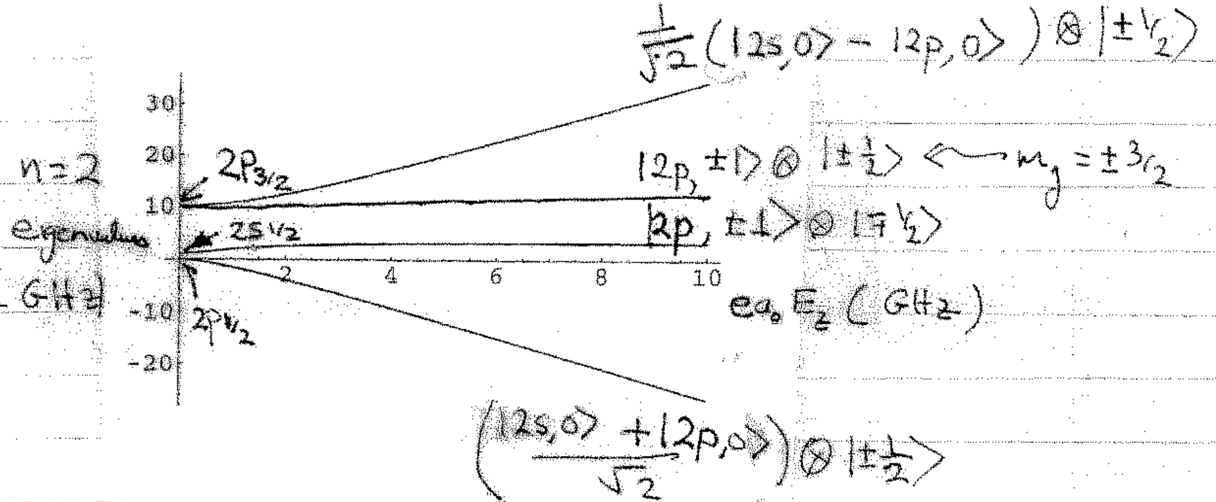
$$= -eE_z \left[\sqrt{\tfrac{2}{3}} \underbrace{\langle 2p, 0 | z | 2s\, 0 \rangle}_{-3a_0} \underbrace{\langle \tfrac{1}{2} | \tfrac{1}{2} \rangle}_{=1} + \sqrt{\tfrac{1}{3}} \langle 2p, 1 | z | 2s, 0 \rangle \underbrace{\cancel{\langle -\tfrac{1}{2} | \tfrac{1}{2} \rangle}}_{0} \right.$$

$$\Rightarrow \boxed{\beta = \sqrt{6}\, ea_0 E_z}$$

$$\Rightarrow \hat{A} \doteq \Delta E_L \begin{bmatrix} 1 & \sqrt{3}x & \sqrt{6}x \\ \sqrt{3}x & 0 & 0 \\ \sqrt{6}x & 0 & 10 \end{bmatrix} \qquad x \equiv \frac{ea_0 E_z}{\Delta E_L}$$

$\Delta E_L = 1\,\text{GHz}$

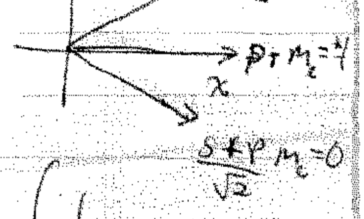
Solving for the eigenvalues numerically in the range $0 < x < 10$



(c) Asymptotic behavior. Note for small x we recover the behavior of part (a) (the level $|2P_{3/2}\rangle$ is too far away). For sufficiently large x the fine-structure is negligible and we recover the simple linear Stark shift discussed in class. That we cover the expected eigenvectors can be seen in the large x limit setting $\frac{\Delta E_{FS}}{x} = \frac{\Delta E_L}{x} = 0$

$$x \gg 1 \Rightarrow \hat{H} \doteq -x \begin{bmatrix} 0 & \sqrt{3} & \sqrt{6} \\ \sqrt{6} & 0 & 0 \\ \sqrt{3} & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ Eigenvalues $\{-3x, 0, 3x\}$



(linear Stark)

Eigenvectors (next page)

$$\begin{array}{l} \text{Eigenvectors:} \\ \left(\begin{array}{l}\text{up to arbitrary} \\ \text{overall phase}\end{array}\right) \end{array} \quad \left(\begin{array}{ccc} |e_1\rangle & |e_2\rangle & |e_3\rangle \\ \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}, & \begin{bmatrix} 0 \\ -\sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}, & \begin{bmatrix} +\sqrt{\frac{1}{2}} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} \end{array} \right)$$

in the ordered basis $(|2s_{1/2}\rangle, |2p_{1/2}\rangle, |2p_{3/2}\rangle)$ $m_J = \frac{1}{2}$

$$\Rightarrow |e_1\rangle = -\frac{1}{\sqrt{2}}|2s_{1/2}\rangle + \frac{1}{\sqrt{6}}|2p_{1/2}\rangle + \frac{1}{\sqrt{3}}|2p_{3/2}\rangle$$

$$= -\frac{1}{\sqrt{2}}|2s,0\rangle|\tfrac{1}{2}\rangle + \frac{1}{3\sqrt{2}}|2p,0\rangle|\tfrac{1}{2}\rangle - \cancel{\frac{1}{3}|2p,1\rangle|-\tfrac{1}{2}\rangle}$$
$$+ \frac{2}{3\sqrt{2}}|2p,0\rangle|\tfrac{1}{2}\rangle + \cancel{\frac{1}{3}|2p,1\rangle|-\tfrac{1}{2}\rangle}$$

$$\boxed{|e_1\rangle = -\frac{1}{\sqrt{2}}\left(|2s,0\rangle - |2p,0\rangle\right) \otimes |\tfrac{1}{2}\rangle}$$

$$|e_2\rangle = -\sqrt{\frac{2}{3}}|2p_{1/2}\rangle + \frac{1}{\sqrt{3}}|2p_{3/2}\rangle = -\frac{\sqrt{2}}{3}|2p,0\rangle|\tfrac{1}{2}\rangle + \frac{2}{3}|2p,1\rangle|-\tfrac{1}{2}\rangle$$
$$+ \frac{\sqrt{2}}{3}|2p,0\rangle|\tfrac{1}{2}\rangle + \frac{1}{3}|2p,1\rangle|-\tfrac{1}{2}\rangle$$

$$\boxed{|e_2\rangle = |2p,1\rangle \otimes |-\tfrac{1}{2}\rangle}$$

Same procedure $\Rightarrow$ $\boxed{|e_3\rangle = \frac{1}{\sqrt{2}}\left(|2s,0\rangle + |2p,0\rangle\right) \otimes |\tfrac{1}{2}\rangle}$

Note the $m_J = -1/2$ are the same asymptotes @ $m_s \to -m_s$

the $m_J = -3/2$ asymptote are flat throughout

and yeild the remaining states $|2p,\pm 1\rangle \otimes |\pm\tfrac{1}{2}\rangle$

Physics 531: Problem Set 3 Solutions

Problem 2: The Zeeman Shift

In the ground state of hydrogenic atoms

$$\hat{H}_{int} = A\vec{I}\cdot\vec{S} - \vec{\mu}_{ep}\cdot\vec{B} - \vec{\mu}_{nuc}\cdot\vec{B}$$

For Hydrogen $\mu_B \gg \mu_N$ $\quad \mu_B = \frac{e\hbar}{2m_e c}$, $\mu_N = \frac{e\hbar}{2m_p c}$

$$\Rightarrow \quad \hat{H}_{int} \cong A\vec{I}\cdot\vec{S} + \underset{\substack{\uparrow \\ \text{g-factor}}}{2}\mu_B B S_z$$

(a) We seek to diagonalize $\hat{H}_{int}$ in the 4D subspace of relative motion and electron/proton spins. This operator in neither diagonal in the coupled or uncoupled spin basis, so we must choose one and then explicitly diagonalize the matrix. Here I will choose to express $\hat{H}_{int}$ in the coupled basis

$\{|F, M_F\rangle\}$ $\quad \left\{\begin{matrix} F=0 \\ M_F=0 \end{matrix}\right\}\{F=1, M_F = 1, 0, -1\}$

$$|F=0, M_F=0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_e|\downarrow\rangle_p - |\downarrow\rangle_e|\uparrow\rangle_p\right)$$

$$|F=1, M_F=1\rangle = |\uparrow\rangle_e|\uparrow\rangle_p$$

$$|F=1, M_F=0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_e|\downarrow\rangle_p + |\downarrow\rangle_e|\uparrow\rangle_p\right)$$

$$|F=1, M_F-1\rangle = |\downarrow\rangle_e|\downarrow\rangle_p$$

Now $\vec{I}\cdot\vec{S}$ is diagonal in this basis

$$\vec{I}\cdot\vec{S} = \frac{1}{2}\left(F(F+1) - \frac{3}{2}\right)$$

But S_z is not. So, consider

$$\langle F' M_F' | \hat{S}_z | F M_F \rangle$$

Note $\hat{S}_z = \hat{S}_z \otimes \mathbb{1}_{Nuc}$ ← Identity on nucleus

Furthermore $[\hat{S}_z, \hat{F}_z] = 0 \Rightarrow$ No off-diagonal matrix elements between different M_F

Diagonal elements of $\hat{S}_z$

$$\langle F=0, M_F=0 | \hat{S}_z | F=0, M_F=0 \rangle = \frac{1}{2}\Big(\underbrace{\langle \uparrow_e | \hat{S}_z | \uparrow_e \rangle}_{=+\frac{1}{2}} \underbrace{\langle \downarrow_p | \downarrow_p \rangle}_{=1} + \underbrace{\langle \downarrow_e | \hat{S}_z | \downarrow_e \rangle}_{=-\frac{1}{2}} \underbrace{\langle \uparrow_p | \uparrow_p \rangle}_{=1}$$

$$- \langle \uparrow_e | \hat{S}_z | \downarrow_e \rangle \underbrace{\langle \downarrow_p | \uparrow_p \rangle}_{0 \text{ orthogonal}} - \langle \downarrow_e | \hat{S}_z | \uparrow_e \rangle \underbrace{\langle \uparrow_p | \downarrow_p \rangle}_{0} \Big)$$

$$= \frac{1}{2}\left(\frac{1}{2} - \frac{1}{2}\right) = 0$$

$$\langle F=1, M_F=1 | \hat{S}_z | F=1, M_F=1 \rangle = \langle \uparrow_e | \hat{S}_z | \uparrow_e \rangle \langle \uparrow_p | \uparrow_p \rangle = +\frac{1}{2}$$

$$\langle F=1, M_F=-1 | \hat{S}_z | F=1\ M_F=-1 \rangle = \langle \downarrow_e | \hat{S}_z | \downarrow_e \rangle \langle \downarrow_p | \downarrow_p \rangle = -\frac{1}{2}$$

$$\langle F=1, M_F=0 | \hat{S}_z | F=1, M_F=0 \rangle = 0 \text{ by same argument}$$

Off-diagonal elements of $\hat{S}_z$

$$\langle F=0, M_F=0| \hat{S}_z | F=1, M_F=0\rangle$$

$$= \frac{1}{2}\left(\langle\uparrow_e|\langle\downarrow_p| - \langle\downarrow_e|\langle\uparrow_p|\right) \hat{S}_z \left(|\uparrow_e\rangle|\downarrow_p\rangle + |\downarrow_e\rangle|\uparrow_p\rangle\right)$$

$$= \frac{1}{2}\left(\langle\uparrow_e|\langle\downarrow_p| - \langle\downarrow_e|\langle\uparrow_p|\right)\left(+\frac{1}{2}|\uparrow_e\rangle|\downarrow_p\rangle - \frac{1}{2}|\downarrow_e\rangle|\uparrow_p\rangle\right)$$

$$= \frac{1}{2}$$

Putting it all together,

$$A\vec{I}\cdot\vec{S} = A \begin{array}{c} \begin{array}{cccc} |F=0, M_F=0\rangle & |F=1, M_F=0\rangle & |F=1\ M_F=1\rangle & |F=1, M_F=-1\rangle \end{array} \\ \left[\begin{array}{cc|cc} -3/4 & & & \\ & \frac{1}{4} & & 0 \\ \hline & & \frac{1}{4} & \\ 0 & & & \frac{1}{4} \end{array}\right] \end{array}$$

$$2\mu_B B\ \hat{S}_z = 2\mu_B B \left[\begin{array}{cc|cc} 0 & \frac{1}{2} & & \\ \frac{1}{2} & 0 & & 0 \\ \hline & & \frac{1}{2} & \\ 0 & & & -\frac{1}{2} \end{array}\right]$$

Note: I have ordered the basis so that to $|F=0\ M_F=0\rangle$ and $|F=1, M_F=0\rangle$ are next to each other

We must therefore diagonalize the 2X2 matrix

$$\hat{H}_{int} \doteq \begin{array}{cc} \begin{bmatrix} -\frac{3A}{4} & \mu_B B \\ \mu_B B & \frac{A}{4} \end{bmatrix} & \begin{array}{l} |F=0,\ M_F=0\rangle \\ |F=1,\ M_F=0\rangle \end{array} \end{array}$$

$$= -\frac{A}{4}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{A}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \mu_B B\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(Decomposed into Pauli matrices)

Solution: Eigenvalues $E_\pm = -\frac{A}{4} \pm \sqrt{\frac{A^2}{4} + (\mu_B B)^2}$

Eigenvectors $|+\rangle = \cos\frac{\theta}{2}|F=0, M_F=0\rangle + \sin\frac{\theta}{2}|F=1, M_F=0\rangle$

$|-\rangle = \sin\frac{\theta}{2}|F=0, M_F=0\rangle - \cos\frac{\theta}{2}|F=1, M_F=0\rangle$

where $\tan\theta = \dfrac{\mu_B B}{-A/2} \Rightarrow \theta = \pi - \tan^{-1}\left(\dfrac{2\mu_B B}{A}\right)$

Note limits • $\mu_B B \ll \frac{A}{2} \Rightarrow \theta \to \pi$

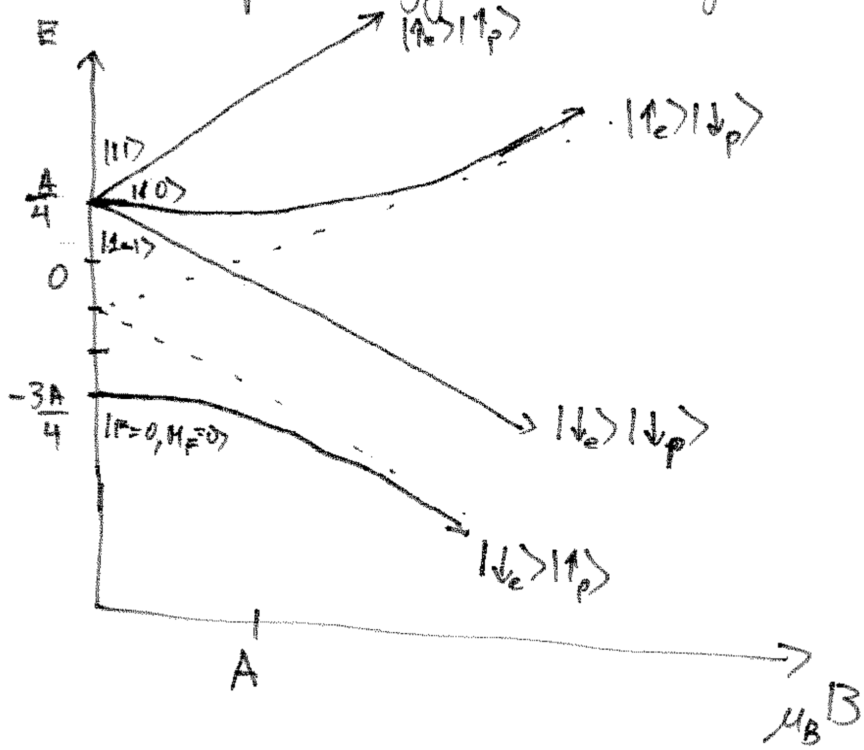
$E_+ \to \frac{A}{4}$ $\quad E_- \to -\frac{3A}{4}$

$|+\rangle \to |F=1, M_F=0\rangle$ $\quad |-\rangle \longrightarrow |F=0,\ M_F=0\rangle$

• $\mu_B B \gg \frac{A}{2} \Rightarrow \theta \to \pi/2$ $\quad E_\pm \to \pm\mu_B B$

$|+\rangle \Rightarrow |\uparrow_e\rangle|\downarrow_p\rangle$ $\quad |-\rangle \Rightarrow |\downarrow_e\rangle|\uparrow_p\rangle$

Sketch of energy level diagram



<u>Key points:</u>

- For small magnetic fields ($\mu_B B \ll \frac{A}{2}$) F, M_F are approximate "good quantum numbers". We then see a "linear Zeeman shifts" proportional to M_F

- For large magnetic fields, the electron and proton spins <u>decouple</u>. Then $|S M_s\rangle|I M_I\rangle$ are approximate good quantum numbers. This is known as the "Paschen-Back" regime.

(b) We consider now positronium and muonium. We can no longer neglect the "nucleus" interaction with the external magnetic field.

$$-\vec{\mu}_{lep}\cdot\vec{B} - \vec{\mu}_{nuc}\cdot\vec{B} = \underbrace{g_{lep}\,\mu_{lep}B}_{\equiv \hbar\omega_{lep}}\,\hat{S}_z - \underbrace{g_{nuc}\,\mu_{nuc}B}_{\hbar\omega_{nuc}}\,\hat{I}_z \equiv \hat{H}_B$$

Let us act $\hat{H}_B$ on the 4-basis states

$$\hat{H}_B|F=0, M_F=0\rangle = \frac{1}{\sqrt{2}}\hbar\omega_{lep}\left\{\hat{S}_z|\uparrow_e\rangle\otimes|\downarrow_p\rangle - \hat{S}_z|\downarrow_e\rangle\otimes|\uparrow_p\rangle\right\}$$

$$-\frac{1}{\sqrt{2}}\hbar\omega_{nuc}\left\{|\uparrow_e\rangle\otimes\hat{I}_z|\downarrow_p\rangle - |\downarrow_e\rangle\otimes\hat{I}_z|\uparrow_p\rangle\right\}$$

$$= \hbar\left(\frac{\omega_{lep}+\omega_{nuc}}{2}\right)\left\{\frac{|\uparrow_e\downarrow_p\rangle + |\downarrow_e\uparrow_p\rangle}{\sqrt{2}}\right\} = \hbar\left(\frac{\omega_{lep}+\omega_{nuc}}{2}\right)|F=1, M_F=1\rangle$$

$$\hat{H}_B|F=1, M_F=0\rangle = \hbar\left(\frac{\omega_{lep}+\omega_{nuc}}{2}\right)|F=0, M_F=0\rangle$$

$$\hat{H}_B|F=1, M_F=1\rangle = \hbar\omega_{lep}\,\hat{S}_z|\uparrow_e\rangle\otimes|\uparrow_p\rangle - \hbar\omega_{nuc}|\uparrow_e\rangle\otimes\hat{I}_z|\uparrow_p\rangle$$

$$= \hbar\left(\frac{\omega_{lep}-\omega_{nuc}}{2}\right)|F=1, M_F=1\rangle$$

$$\hat{H}_B|F=1, M_F-1\rangle = -\hbar\left(\frac{\omega_{lep}-\omega_{nuc}}{2}\right)|F=1, M_F=-1\rangle$$

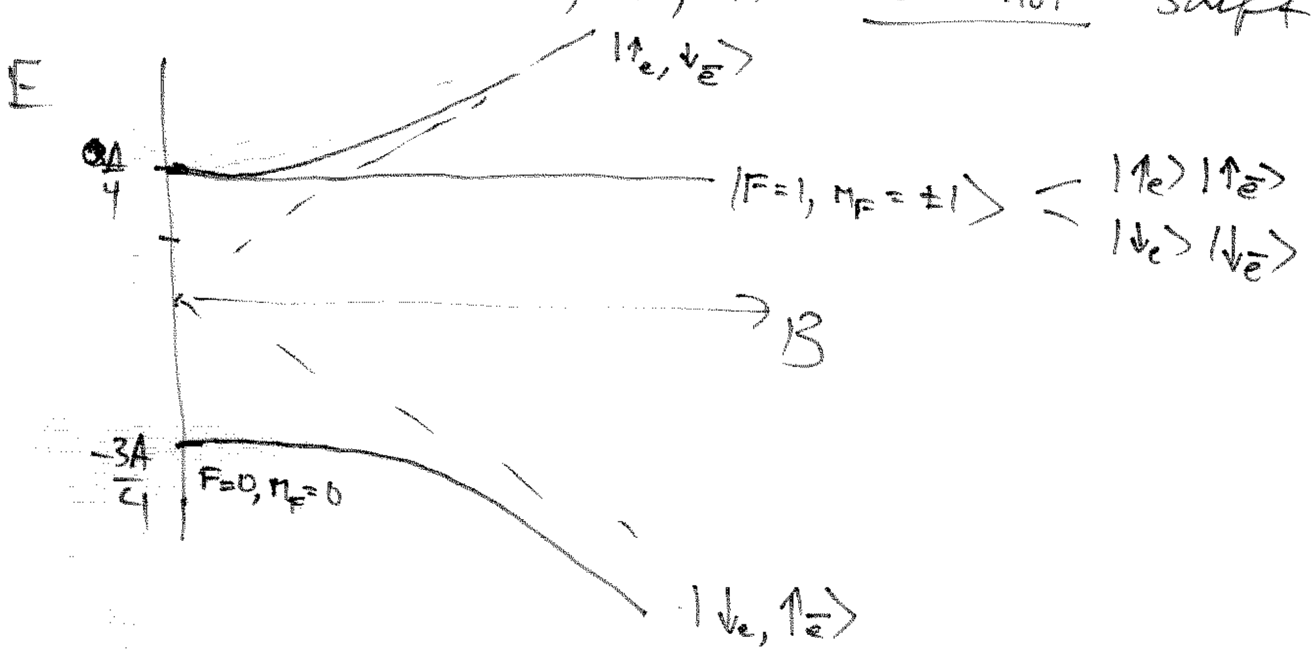
<u>Positronium:</u> $\left.\begin{array}{l} g_{lep} = g_{nuc} = 2 \\ \mu_{lep} = \mu_{nuc} = \mu_B \end{array}\right\} \Rightarrow$ $\hbar\omega_{lep} = \hbar\omega_{nuc} = 2\mu_B B$

$$\Rightarrow \hat{H}_{int} = \begin{array}{c} \begin{array}{cccc} |00\rangle & |10\rangle & |11\rangle & |1\,{-1}\rangle \end{array} \\ \left[\begin{array}{cc|cc} -\frac{3A}{2} & 2\mu_B B & \multicolumn{2}{c}{0} \\ 2\mu_B B & \frac{A}{4} & & \\ \hline \multicolumn{2}{c|}{0} & \frac{A}{4} & 0 \\ & & 0 & \frac{A}{4} \end{array}\right] \end{array}$$

$\Rightarrow$ New eigenvalues: $E_\pm = -\frac{A}{4} \pm \sqrt{\left(\frac{A}{2}\right)^2 + (2\mu B)^2}$

eigenvectors
$$|+\rangle = \cos\tfrac{\theta}{2}\,|0,0\rangle + \sin\tfrac{\theta}{2}\,|1,0\rangle$$
$$|-\rangle = \sin\tfrac{\theta}{2}\,|0,0\rangle - \cos\tfrac{\theta}{2}\,|1,0\rangle$$
$$\tan\theta = \frac{2\mu_B B}{-A/2}$$

the states $|11\rangle$, $|1,-1\rangle$ <u>do not</u> shift



Muonium:

$$g_{lep} = 2 \qquad g_{nuc} = 5.9$$

$$\mu_{lep} = \frac{e\hbar}{2m_\mu c} \qquad \mu_{nuc} = \frac{e\hbar}{2m_p c}$$

$$\Rightarrow \hbar\omega_{lep} = 2\mu_{lep}B = \left(\frac{m_e}{m_\mu}\right) 2\mu_B B$$

$$\hbar\omega_{nuc} = 5.9\,\mu_{nuc}B = \left(\frac{m_e}{m_p}\right)(5.9\,\mu_B B)$$

$$\Rightarrow E_\pm = -\frac{A}{4} \pm \sqrt{\left(\frac{\hbar}{2}(\omega_{lep}+\omega_{nuc})\right)^2 + \left(\frac{A}{2}\right)^2} \longrightarrow \pm\frac{\hbar}{2}(\omega_{lep}+\omega_{nuc})$$

For larger field

$$E_{11,\pm 1} = \pm\frac{\hbar}{2}(\omega_{lep}-\omega_{nuc})$$

thus, the $E_\pm$ states have a different slope for large fields