Physics 531

Problem Set #7: Solutions

Problem 1: Argon spectrum

(a) Low Pressure (Ar I)

	Line (Å)	Initial	Final	ΔJ	Single orbital
(1)	9123.0	$(3p)^5(4p)$: $^3S_1$	$(3p)^5(4s)$: $^3P_2$	+1	$4p \to 4s$
(2)	8115.3	$(3p)^5(4p)$: $^3D_3$	$(3p)^5(4s)$: $^3P_2$	−1	$4p \to 4s$
(3)	7384.0	$(3p)^5(4p)$: $^3P_2$	$(3p)^5(4s)$: $^3P_1$	−1	$4p \to 4s$
(4)	7067.2	$(3p)^5(4p)$: $^3P_2$	$(3p)^5(4s)$: $^3P_2$	0	$4p \to 4s$
(5)	6965.2	$(3p)^5(4p)$: $^3P_1$	$(3p)^5(4s)$: $^3P_2$	+1	$4p \to 4s$
(6)	4200.7	$(3p)^5(5p)$: $^3D_3$	$(3p)^5(4s)$: $^3P_2$	+1	$5p \to 4s$

All transitions electric dipole, triplet → triplet

All involve a change in parity since $\Delta l = \pm 1$

Note: the NIST database uses a different coupling scheme to denote the term. For example, they list the ground triplet state as

$$(3p)^5\,(^2P_{3/2})\,4s \;:\; ^2[3/2]_2$$

This is known as $J_1 - l_2$ coupling between two open sub-shells, one with multiple equivalent electrons, the other with one electron (see http://physics.nist.gov/Pubs/AtSpec/index.html)

We can, instead, denote the terms using Russell-Saunders L-S coupling

The config $(3p)^5(^2P_{3/2})\,4s \Rightarrow$ add $\left.\begin{array}{l} L_1 = 1 \\ L_2 = 0 \end{array}\right\} \Rightarrow L = 1$

All terms are triplets since they involve the the possibility of $J_{total} = 2$ $(^2P_{3/2} + 4s)$

$\Rightarrow$ Ground triplet ^3P

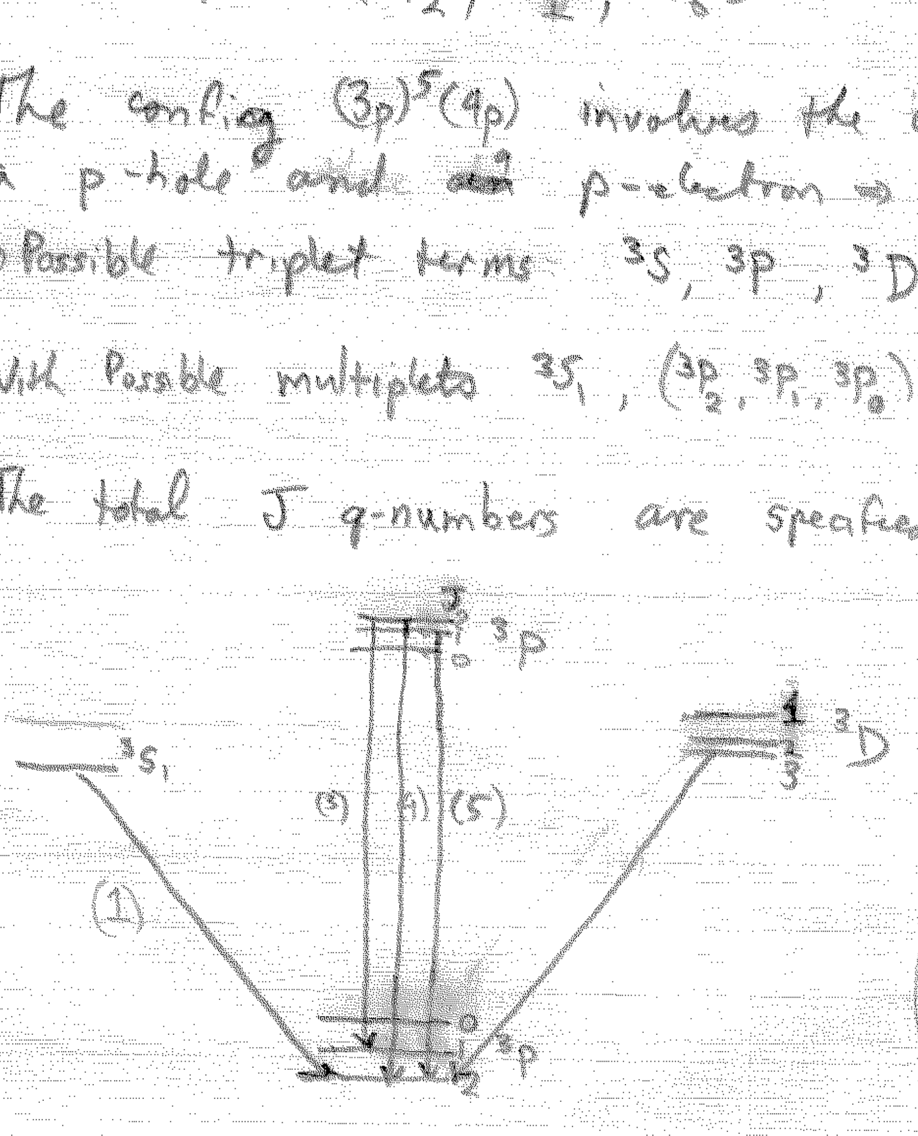
multiplet $\{^3P_2, {}^3P_1, {}^3P_0\}$

The config $(3p)^5(4p)$ involves the coupling of a p-hole and a p-electron $\Rightarrow L = 0, 1, 2$

$\Rightarrow$ Possible triplet terms ^3S, ^3P, ^3D

With possible multiplets $^3S_1$, $(^3P_2, {}^3P_1, {}^3P_0)$, $(^3D_3, {}^3D_2, {}^3D_1)$

The total J q-numbers are specified in the NIST data



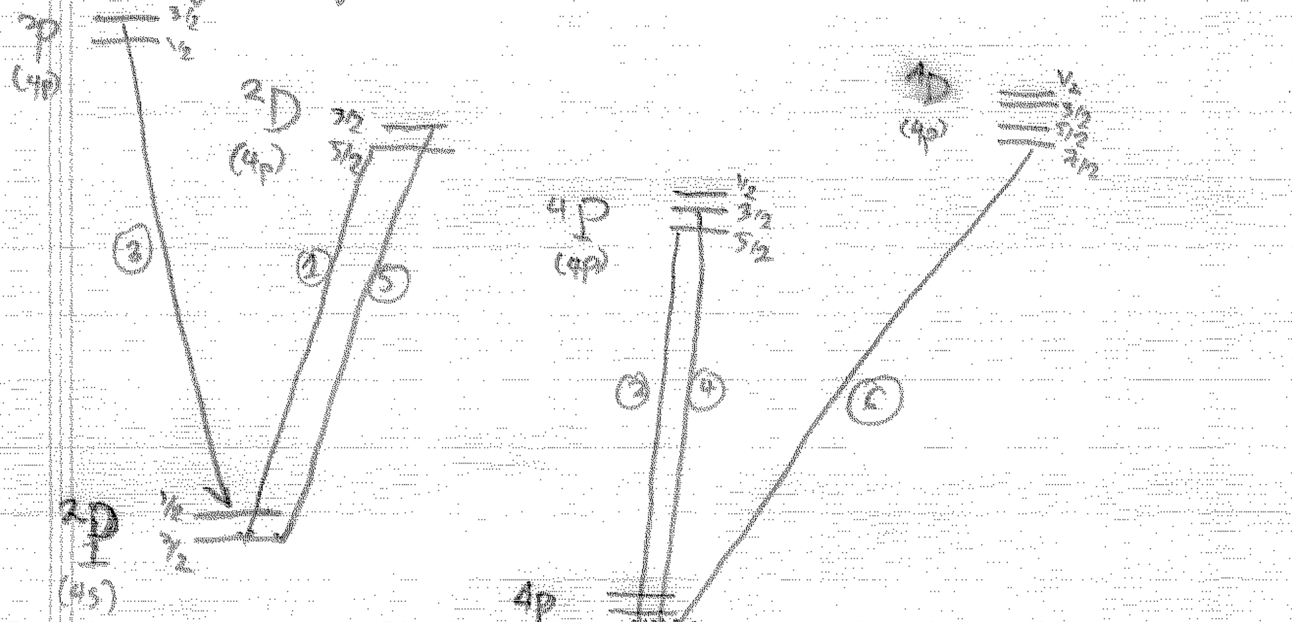
Not shown: Highly excited $(3p)^5(5p)$: $^3D_3$

(b) High pressure (Ar II)

	Line (Å)	Initial	Final	ΔJ	Single-orbital
(1)	4879.9	$(3p)^4 4p: {}^2D_{5/2}$	$(3p)^4 4s: {}^2P_{3/2}$	-1	$4p \to 4s$
(2)	4806.0	$(3p)^4 4p: {}^4P_{5/2}$	$(3p)^4 4s: {}^4P_{5/2}$	0	$4p \to 4s$
(3)	4764.9	$(3p)^4 4p: {}^2P_{3/2}$	$(3p)^4 4s: {}^2P_{1/2}$	-1	$4p \to 4s$
(4)	4735.9	$(3p)^4 4p: {}^4P_{3/2}$	$(3p)^4 4s: {}^4P_{5/2}$	$+1$	$4p \to 4s$
(5)	4726.9	$(3p)^4 4p: {}^2D_{3/2}$	$(3p)^4 4s: {}^2P_{3/2}$	0	$4p \to 4s$
(6)	4348.1	$(3p)^4 4p: {}^4D_{7/2}$	$(3p)^4 4s: {}^4P_{5/2}$	-1	$4p \to 4s$

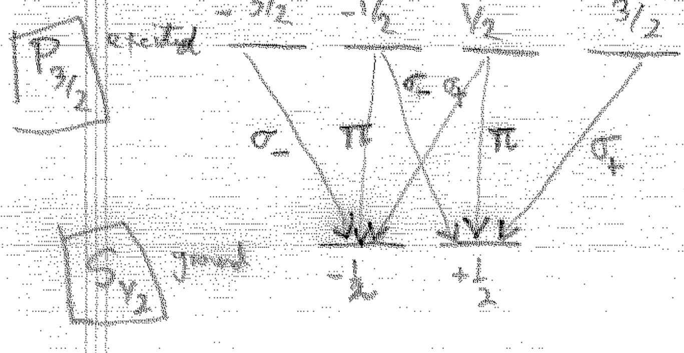
All transitions are electric dipole allowed $\Delta J = 0, \pm 1$, $\Delta S = 0$

Parity change since $\Delta l = 1$ (single particle orbital)



Problem 2: The Zeeman Effect

D2 line of an alkali $P_{3/2} \rightarrow S_{1/2}$



The six allowed transition (E1) with $\Delta m_J = 0, \pm 1$ are shown
$\Delta m_J = 0$ (π-polarization)
$\Delta m_J = m_J(P_{3/2}) - m_J(P_{1/2}) = \pm 1$
$\Rightarrow$ ($\sigma_\pm$-polarization)

The effect of the magnetic field is described by Hamiltonian

$$\hat{H}_{int} = -\vec{\mu} \cdot \vec{B}$$

By the Landé-Projection theorem previously studied

$$\vec{\mu} = -g_J \mu_B \vec{J} \quad \text{(in units of } \hbar\text{)}$$

Define $\vec{B} = B\vec{e}_z$ (the quantization axis along z)

$$\Rightarrow \hat{H}_{int} = g_J(\mu_B B)\hat{J}_z \qquad g_J = \frac{3}{2} + \frac{s(s+1) - \ell(\ell+1)}{2J(J+1)}$$

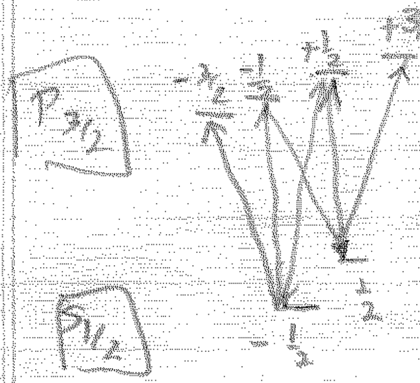
$\Rightarrow$ $S_{1/2}$ $(\ell = 0,\ s = 1/2,\ J = 1/2)$ $\Rightarrow$ $g_{J=1/2} = 2$

$P_{3/2}$ $(\ell = 1,\ s = 1/2,\ J = 3/2)$ $\Rightarrow$ $g_{J=3/2} = \frac{4}{3}$

$\Rightarrow$ Perturbation break degeneracy of levels

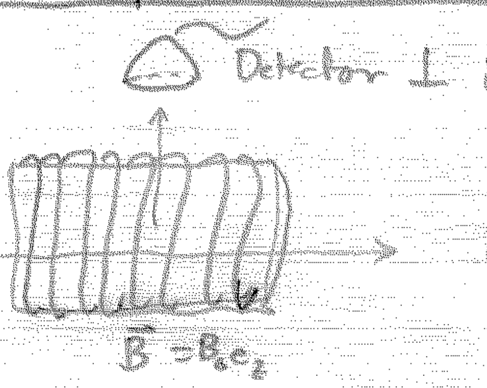
$$\Delta E = g_J(\mu_B B)\, m_J$$

Energy levels with B-field



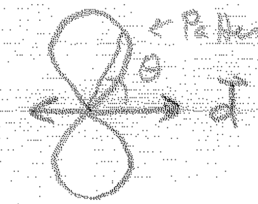
- 6 lines of different frequencies
- Not 8 lines because $\Delta m_J = \pm 2$ forbidden

<u>Now lets think about the geometry.</u>



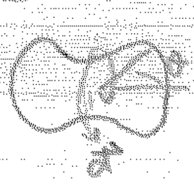
The π, $\sigma_\pm$ light are <u>not</u> emitted isotropically, but according to dipole radiation patterns:

- <u>π polarization:</u> Dipole oscillating linearly along z-axis



$\Rightarrow$ No π-light along z

- <u>$\sigma_\pm$ polarization:</u> Dipole rotating about z-axis



Pattern $\frac{1}{2}(1+\cos^2\theta)$

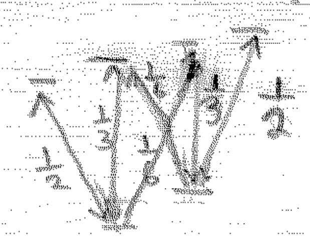
Twice as much emitted along z and ⊥

(b) The relative intensities depend on the squares of the dipole matrix elements

$$|\langle P_{1/2}\ m_J' |\ d_q\ | P_{3/2}\ m_J \rangle|^2 = |\langle P_{1/2} \| d \| P_{3/2} \rangle|^2$$

$$|\langle \tfrac{1}{2}\ m_J' |\ 1\ q\ \ \tfrac{3}{2}\ m_J \rangle|^2$$

Square of C.G. coeff



denoted here for allowed transitions

The Relative intensities as a function of θ

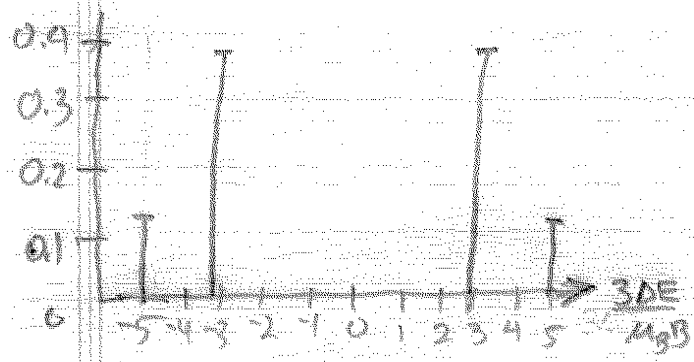
$$I_{m_J' \to m_J} = \begin{cases} \sigma_\pm : \tfrac{1}{2}(1+\cos^2\theta)\, |\langle \tfrac{3}{2}\ m_J \mp 1 |\ 1\ q = \mp 1, \tfrac{3}{2}\ m_J \rangle|^2 \\ \pi : \sin^2\theta\, |\langle \tfrac{3}{2}\ m_J |\ 1\ q=0, \tfrac{1}{2}\ m_J \rangle|^2 \end{cases}$$

Below is a table of the six allowed transitions, there energy differenCe compare to $P_{3/2} \to S_{1/2}$

$\Delta E = (\tfrac{4}{3} m_J' - 2 m_J)\, \mu_B B$, and the relative intensities

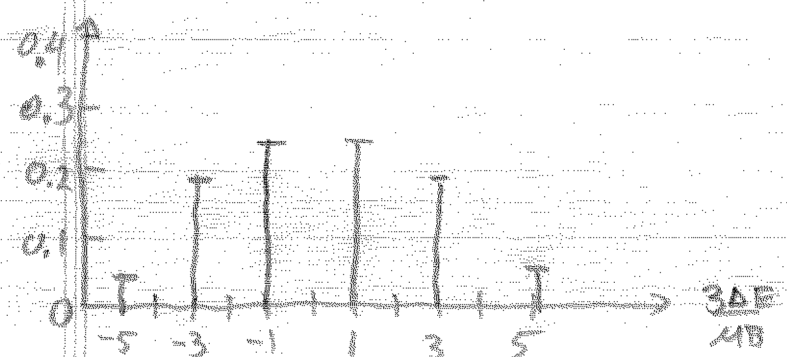
$P_{3/2}$ m_J	$S_{1/2}$ m_J'	pol	$\Delta E / \mu_B B$	I	$I_\parallel^{\theta=0}$	$I_\perp^{\theta=\frac{\pi}{2}}$
$\frac{3}{2}$	$\frac{1}{2}$	σ_+	1	$\frac{1}{4}(1+\cos^2\theta)$	$\frac{1}{2}$	$\frac{1}{4}$
$\frac{1}{2}$	$\frac{1}{2}$	π	$-\frac{1}{3}$	$\frac{1}{3}\sin^2\theta$	0	$\frac{1}{3}$
$\frac{1}{2}$	$-\frac{1}{2}$	σ_+	$5/3$	$\frac{1}{12}(1+\cos^2\theta)$	$\frac{1}{6}$	$\frac{1}{12}$
$-\frac{1}{2}$	$\frac{1}{2}$	σ_-	$-5/3$	$\frac{1}{12}(1+\cos^2\theta)$	$\frac{1}{6}$	$\frac{1}{12}$
$-\frac{1}{2}$	$-\frac{1}{2}$	π	$\frac{1}{3}$	$\frac{1}{3}\sin^2\theta$	0	$\frac{1}{3}$
$-\frac{3}{2}$	$-\frac{1}{2}$	σ_-	-1	$\frac{1}{4}(1+\cos^2\theta)$	$\frac{1}{2}$	$\frac{1}{4}$

The normalized intensity patterns for polarization in singlet line



$\theta = 0$

(no π component)



$\theta = \frac{\pi}{2}$

all six allowed

(c) For detector perpendicular the transitions are all polarized, thus we ~~see~~ will all six transitions label above. The bar chart is the same