## Physics 33.1

### Problem Set 7: Solutions

#### Problem 1: Argon spectrum

(a) Low Pressure (Ar I)

<table>
<thead>
<tr>
<th>Line (Å)</th>
<th>Initial</th>
<th>Final</th>
<th>( \Delta J )</th>
<th>Singlet orbital</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) 9123.0</td>
<td>(3p(^5)) (4p(^3)) ( ^3S_1 )</td>
<td>(3p(^5)) (3p(^3)) ( ^3P_2 )</td>
<td>+1</td>
<td>1p ( \rightarrow ) 4s</td>
</tr>
<tr>
<td>(2) 9115.3</td>
<td>(3p(^5)) (4p(^3)) ( ^3D_3 )</td>
<td>(3p(^5)) (4p(^3)) ( ^3P_2 )</td>
<td>-1</td>
<td>4p ( \rightarrow ) 4s</td>
</tr>
<tr>
<td>(3) 9384.0</td>
<td>(3p(^5)) (4p(^3)) ( ^3P_2 )</td>
<td>(3p(^5)) (3p(^3)) ( ^3P_1 )</td>
<td>-1</td>
<td>4p ( \rightarrow ) 4s</td>
</tr>
<tr>
<td>(4) 7067.2</td>
<td>(3p(^5)) (4p(^3)) ( ^3P_2 )</td>
<td>(3p(^5)) (3p(^3)) ( ^3P_2 )</td>
<td>0</td>
<td>4p ( \rightarrow ) 4s</td>
</tr>
<tr>
<td>(5) 6965.2</td>
<td>(3p(^5)) (4p(^3)) ( ^3P_1 )</td>
<td>(3p(^5)) (3p(^3)) ( ^3P_2 )</td>
<td>+1</td>
<td>4p ( \rightarrow ) 4s</td>
</tr>
<tr>
<td>(6) 4200.7</td>
<td>(3p(^5)) (3p(^3)) ( ^3D_3 )</td>
<td>(3p(^5)) (3p(^3)) ( ^3P_2 )</td>
<td>+1</td>
<td>5p ( \rightarrow ) 4s</td>
</tr>
</tbody>
</table>

All transitions are electric dipole, triplet \( \rightarrow \) triplet.

All involve a change in parity since \( \Delta L = \pm 1 \)

**Note:** The NIST database uses a different coupling scheme to denote the term. For example, they list the ground triplet state as

\[ (3p)^5 \ (2P_{3/2}) \ 4s : \ 2^3[3/2] \]

This is known as \( J \)-\( L \) coupling between two open sub-shells, one with multiple equivalent electrons, the other with one electron (see [http://physics.nist.gov/Pubs/AtSpec/index.html](http://physics.nist.gov/Pubs/AtSpec/index.html)).
We can, instead, denote the terms using Russell-Saunders $L-S$ coupling.

The config. $(3p)^5 (2p_{3/2}) 4s \Rightarrow \text{Add } L = 1, 3 \Rightarrow L = 1,
\begin{align*}
L^2 = 0, 3 \Rightarrow L = 1,
\end{align*}

All terms are triplets since they involve the possibility of $J_{total} = 2$ ($2p_{3/2} + 4s$)

$\Rightarrow \text{Ground triplet } \frac{3}{2}p
\text{ multiplet } \{ \frac{3}{2}p_2, \frac{3}{2}p_1, \frac{3}{2}p_0 \}

The config. $(3p)^5 (4p)$ involves the coupling of a $p$-hole and a $p$-electron $L = 0, 1, 2$

$\Rightarrow \text{Possible triplet terms } \frac{3}{2}s, \frac{3}{2}p, \frac{3}{2}d$

With possible multiplets $\frac{3}{2}s, (\frac{3}{2}p_2, \frac{3}{2}p_1, \frac{3}{2}p_0), (\frac{3}{2}d_3, \frac{3}{2}d_2, \frac{3}{2}d_1)$

The total $J$-numbers are specified in the MIST data.

\[
\begin{array}{c}
3s_1 \\
(1) \\
(3) \\
(5)
\end{array}
\]

\[
\begin{array}{c}
3p \\
\frac{1}{3} 3d
\end{array}
\]

(Note shown:
Highly excited
$(2p)^5 (5p): \frac{3}{2}d_3$)
(b) High pressure (Ar II)

<table>
<thead>
<tr>
<th>Line (Å)</th>
<th>Initial</th>
<th>Final</th>
<th>ΔJ</th>
<th>Single-allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) 4874.9</td>
<td>(3p)⁴4p: ²D⁹/₂</td>
<td>(3p)⁷4s: ²P⁹/₂</td>
<td>-1</td>
<td>4p → 4s</td>
</tr>
<tr>
<td>(2) 4806.0</td>
<td>(3p)⁴4p: ⁴P⁹/₂</td>
<td>(3p)⁷4s: ⁴P⁹/₂</td>
<td>0</td>
<td>4p → 4s</td>
</tr>
<tr>
<td>(3) 4764.9</td>
<td>(3p)⁴4p: ⁴P³/₂</td>
<td>(3p)⁷4s: ⁴P⁹/₂</td>
<td>-1</td>
<td>4p → 4s</td>
</tr>
<tr>
<td>(4) 4735.9</td>
<td>(3p)⁴4p: ⁴P⁹/₂</td>
<td>(3p)⁷4s: ⁴P⁹/₂</td>
<td>1</td>
<td>4p → 4s</td>
</tr>
<tr>
<td>(5) 4726.9</td>
<td>(3p)⁴4p: ⁴D⁹/₂</td>
<td>(3p)⁷4s: ⁴P⁹/₂</td>
<td>0</td>
<td>4p → 4s</td>
</tr>
<tr>
<td>(6) 4349.1</td>
<td>(3p)⁴4p: ⁴D⁹/₂</td>
<td>(3p)⁷4s: ⁴P⁹/₂</td>
<td>-1</td>
<td>4p → 4s</td>
</tr>
</tbody>
</table>

All transitions are electric dipole allowed  
ΔS = 0  
ΔJ = 0, ±1

Parity change since Δl = 1 (Single partial orbital)
Problem 2: The Zeeman Effect

D2 line of an alkali \( P_{3/2} \rightarrow S_{1/2} \)

\[ \begin{array}{cccc}
\text{excited} & 3/2 & -1/2 & 1/2 \\
\sigma^- & \pi & \pi & \pi \\
\sigma^+ & \sigma & \sigma^+ & \sigma^+
\end{array} \]

\( \Delta m_J = 0, \pm 1 \) are shown

\( \Delta m_J = 0 \) (\( \pi \)-polarization)

\( \Delta m_J = m_J (P_{3/2}) - m_J (S_{1/2}) = \pm 1 \)

\( \Rightarrow (\sigma^- \text{ or } \sigma^+ \text{ polarization}) \)

The effect of the magnetic field is described by Hamiltonian

\[ \hat{H}_{\text{int}} = -\hat{m} \cdot \overrightarrow{B} \]

By the Landé–Projection Theorem previously studied

\[ \hat{m} = -g_J \mu_B \overrightarrow{J} \] (in units of \( \hbar \))

Define \( \overrightarrow{B} = B \overrightarrow{e}_z \) (the quantization axis along \( z \))

\[ \Rightarrow \hat{H}_{\text{int}} = g_J (\mu_B \overrightarrow{B}) \overrightarrow{J} \cdot \overrightarrow{J} \]

\[ g_J = \frac{\frac{3}{2} + \frac{\sqrt{3} (5J^2 - 3J + 3)}{2J(J+1)}}{\mu_B B} \]

\( S_{1/2} \) (\( l=0, s=1/2, J=1/2 \)) \( g_J \cdot \frac{1}{2} = 2 \)

\( P_{3/2} \) (\( l=1, s=1/2, J=3/2 \)) \( g_J \cdot \frac{3}{2} = \frac{4}{3} \)

\( \Rightarrow \) Perturbation breaks degeneracy of levels

\[ \Delta E = g_J (\mu_B B) m_J \]
Energy levels with B-field

Now, let's think about the geometry.

Detector 1 to $B^+$

Detector 2 to $B^+$

$B = B_0 e_z$

The $\parallel$, $\perp$ light are not emitted isotropically, but according to dipole radiation patterns:

- $\parallel$ polarization: Dipole oscillating linearly along z-axis

  $\theta \rightarrow 0 \rightarrow$ No $\parallel$-light along z

- $\perp$ polarization: Dipole rotating about z-axis

  $\frac{\pi}{2}$ Pattern $\frac{1}{2} (1 + \cos \theta)$ Twice as much emitted along z and y
(b) The relative intensities depend on the squares of the dine matrix elements

\[ |\langle \sigma_{1/2}, m_{\sigma}; d_{3/2} | \sigma_{1/2}, m_{\sigma} \rangle |^2 = |\langle \sigma_{1/2}, m_{\sigma}; d_{3/2} | 1^{-}\rangle |^2 \]

Square of C. O. coeff.

denoted here for allowed transitions

The relative intensities as a function of \( \theta \)

\[ d_{m_{\sigma} \rightarrow m_{\sigma}} = \left( \frac{\lambda}{2} \left( 1 + \cos^2 \theta \right) \right) \left| \langle \frac{3}{2}, m_{\sigma} | 1^{-}\rangle \right|^2 \]

\[ = \sin^2 \theta \left| \langle \frac{1}{2}, m_{\sigma} | 1^{-}\rangle \right|^2 \]

Below is a table of the six allowed transitions, where energy differences compare to \( \sigma_{1/2} \rightarrow \sigma_{1/2} \)

\[ \Delta E = (\frac{3}{2} m_{\sigma}^2 - 2 m_{\sigma}) \mu_B B \], and the relative intensities

<table>
<thead>
<tr>
<th>( \sigma_{1/2} )</th>
<th>( m_{\sigma} )</th>
<th>( \Pi )</th>
<th>( \Delta E/\mu_B B )</th>
<th>( \lambda )</th>
<th>( \lambda_{1^{-}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{2} )</td>
<td>( \frac{3}{2} )</td>
<td>0+</td>
<td>1</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( \pi )</td>
<td>-( \frac{1}{3} )</td>
<td>( \frac{1}{3} ) sin^2 \theta</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( -\frac{1}{2} )</td>
<td>( \sigma^{+} )</td>
<td>( \frac{5}{12} )</td>
<td>( \frac{1}{12} (1 + \cos^2 \theta) )</td>
<td>( \frac{1}{12} )</td>
</tr>
<tr>
<td>( -\frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( \sigma^{-} )</td>
<td>-( \frac{5}{12} )</td>
<td>( \frac{1}{12} (1 + \cos^2 \theta) )</td>
<td>( \frac{1}{12} )</td>
</tr>
<tr>
<td>( -\frac{1}{2} )</td>
<td>( -\frac{1}{2} )</td>
<td>( \pi )</td>
<td>( \frac{1}{3} )</td>
<td>( b \sin^2 \theta )</td>
<td>0</td>
</tr>
<tr>
<td>( -\frac{3}{2} )</td>
<td>( -\frac{1}{2} )</td>
<td>( \sigma^{-} )</td>
<td>-1</td>
<td>( \frac{1}{4} (1 + \cos^2 \theta) )</td>
<td>( \frac{1}{2} )</td>
</tr>
</tbody>
</table>
The normalized intensity patterns for polarization in magnesium.

\[ \theta = 0 \]
(no \( \pi \)-component)

\[ \theta = \frac{\pi}{2} \]
all six allowed

(c) For detector perpendicular, the transitions are all polarized, thus we see all six transitions labeled above. The bar chart is the same.