Physics 566

Lecture 5: Optical Bloch Equation (II) Phenomenological Damping

So far we have considered only "coherent evolution". That is, we ~~ha~~ have considered the dynamics of of two-level systems governed by a simple time dependent Hamiltonian. The resulting evolution exhibited a simple sinusoidal time dependence for all time — it is reversable. In real situations this is never the case — the systems eventually "relax" to some steady state where "memory" of the initial condition is lost. this irreverisble behavior is a fact of nature. How do we take irreverisibility into account in quantum theory whose basic time evolution is a reversible unitary map? This is ~~a~~ a fundamental problem which we will treat in detail in the latter 1/3 of this class. For now we will take a somewhat phenomeno-logical approach. As an aside, the problem of obtaining effective irreversible behavior at the macroscopic level when the microscopic laws of physics are time-reversible is a fundamental issue in classical physics as well; i.e. from Newtonian dynamics to the second law of thermo.

Phenomenological decay:

Consider a two-level system. there are two kinds of decays

- Relaxations of the population ρ_{ee}, ρ_{gg}
- Relaxations of the coherences ρ_{eg}

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In the old NMR notation:

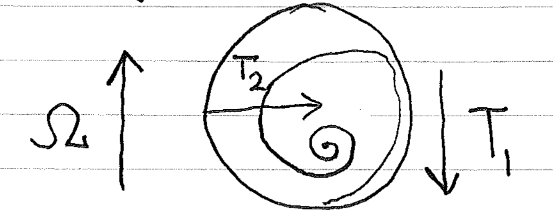
1) $T_1 \equiv$ decay time constant of the population inversion $w \equiv \rho_{ee} - \rho_{gg}$
$\equiv$ "Longitudinal relaxation"

2) $T_2 \equiv$ decay time constant of coherences
$\equiv$ "Transverse relation"

Bloch equations with relaxation

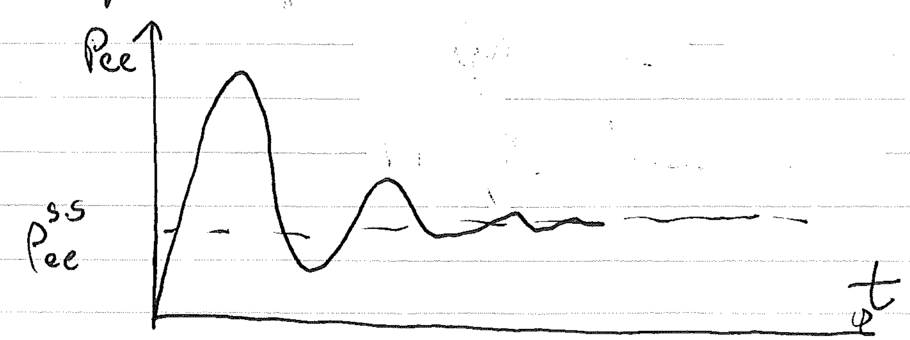
$$\boxed{\begin{aligned} \dot{u} &= -\Delta v - \frac{1}{T_2} u \\ \dot{v} &= \Delta u + \Omega w - \frac{1}{T_2} v \\ \dot{w} &= -\frac{1}{T_1}(w - w_{eq}) - \Omega v \\ & w_{eq} = \text{equilibrium inversion} = -1 \end{aligned}}$$

Trajectory on Bloch sphere



State is no longer described by a pure state.
Must ~~make~~ use density op.

Damped Rabi oscillations



Another picture of "T_1" vs. "T_2"

Consider $\hat{\rho} = \sum_\alpha P_\alpha |\psi_\alpha\rangle\langle\psi_\alpha|$ (ensemble decomposition)

$$\rho_{ij} = \sum_\alpha P_\alpha c_i^\alpha c_j^{\alpha *} = \sum_\alpha P_\alpha |c_i^\alpha||c_j^\alpha| e^{i(\phi_i^\alpha - \phi_j^\alpha)}$$

Diagonal element $\rho_{ii} = \sum_\alpha P_\alpha |c_i^\alpha|^2$

$\Rightarrow$ T_1 involves average relaxation of (magnitude)^2 of probability amplitudes

$$\rho_{ii}(t)\big|_{relax} = e^{-\Gamma_i t} \rho_{ii}(0)$$

Γ_i = decay rate of $\overline{|c^\alpha|^2} \Rightarrow \overline{|c_i^\alpha(t)|} \approx \overline{|c_i^\alpha(0)|}\, e^{-\frac{\Gamma_i}{2} t}$

Off-diagonal, Suppose magnitude same $\forall \alpha$

$$\Rightarrow \rho_{ij}(t) = |c_i(0)||c_j(0)|\, e^{-\left(\frac{\Gamma_i + \Gamma_j}{2}\right)t} \sum_\alpha P_\alpha e^{i(\phi_i^\alpha - \phi_j^\alpha)}$$

Consider process of "dephasing" $\Rightarrow \phi_j^\alpha - \phi_j^\alpha = (\delta\omega_{ij}^\alpha) t$

$$\sum_\alpha P_\alpha e^{i(\delta\omega_{ij}^\alpha)t} \Rightarrow e^{-\beta_{ij} t}$$

$\Rightarrow$ T_2 involves both population change and dephasing

$$\boxed{\frac{1}{T_2} = \frac{\Gamma_i + \Gamma_j}{2} + \beta_{ij} \equiv \gamma_{ij}}$$

Sources of relaxation – Perturbation by the "environment"

e.g. Collisions – elastic $\Rightarrow$ dephasing
inelastic $\Rightarrow$ population change

Generally $T_2 << T_1$

Small changes in phase compared to pop. change.

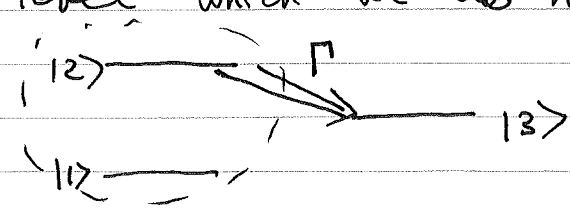
Note: If we have an ensemble of system dephasing can occur due to inhomogeneities (see H.W.). Then
$$\frac{1}{T_2} = \frac{1}{T_2'} + \frac{1}{T_2^*}$$

where $T_2^* =$ decay time due to inhomogeneity
$T_2' =$ decay " for homogeneous sample (truly irreversible)

Unless otherwise stated, we will take $T_2^* = \infty$

Non unitary Schrödinger equation

Consider system with loss of population to extra level which we do not account for



In $\{|1\rangle, |2\rangle\}$ space $\mathrm{Tr}(\rho)$ not conserved

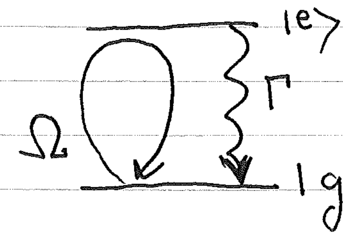
$\Rightarrow$ Simple approx:
Deterministic but non unitary

$$\begin{cases} \dot{c}_1 = -i\frac{E_1}{\hbar} c_1 - i\frac{H_{12}}{\hbar} c_2 \\ \dot{c}_2 = -i\frac{E_2}{\hbar} c_2 - \frac{\Gamma}{2} c_2 - i\frac{H_{21}}{\hbar} c_1 \end{cases}$$

Decaying level $\Rightarrow$ imaginary part of energy

Master equation for two-level atom

Consider a two-level "closed transition"



(population is replenished)

One source of "relaxation": Spontaneous emission

$$\Gamma = \text{"Einstein A-coefficient"} = \frac{1}{\tau} = \frac{4}{3}\frac{|\vec{d}_{eg}|^2}{\hbar}k^3$$

τ = Natural lifetime

$$\dot{\rho}_{ee}\Big|_{relax} = -\Gamma\rho_{ee} = -\frac{1}{T_1}\rho_{ee} \qquad T_1 = \frac{1}{\Gamma} = \tau$$

$$\dot{\rho}_{eg}\Big|_{relax} = -\frac{\Gamma}{2}\rho_{eg} = -\frac{1}{T_2}\rho_{eg} \qquad T_2 = \frac{T_1}{2}$$

Since $\rho_{ee} + \rho_{gg} = 1$ $\qquad \dot{\rho}_{gg}\Big|_{relax} = -\dot{\rho}_{ee}\Big|_{relax} = +\Gamma\rho_{ee}$

Master equation for damped Rabi flopping

$$\boxed{\begin{aligned} \dot{\rho}_{ee} &= -\Gamma\rho_{ee} + i\frac{\Omega}{2}(\rho_{ge} - \rho_{eg}) = -\dot{\rho}_{gg} \\ \dot{\rho}_{ge} &= \left(-i\Delta - \frac{\Gamma}{2}\right)\rho_{ge} + i\frac{\Omega}{2}(\rho_{ee} - \rho_{gg}) \end{aligned}}$$

Same as Bloch e.q.

Operator form:

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] + \mathcal{L}_{relax}[\hat{\rho}]$$

$$\mathcal{L}_{relax}[\hat{\rho}] = \underbrace{-\frac{\Gamma}{2}(\hat{\sigma}_+\hat{\sigma}_-\hat{\rho} + \rho\,\hat{\sigma}_+\hat{\sigma}_-)}_{\text{Decay}} + \underbrace{\Gamma\,\hat{\sigma}_-\hat{\rho}\,\hat{\sigma}_+}_{\text{replenish}}$$