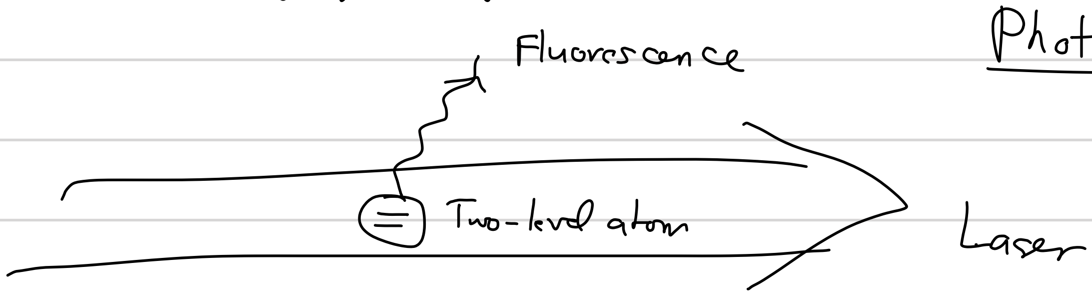
Physics 566 - Quantum Optics 1

Lecture 17: Resonance Fluorescence

We have studied the dynamics of atomic coherence driven by a "classical field" tuned near an atomic resonance. The result is damped Rabi oscillations of the two-level atom. The damping results from "spontaneous emission;" the atom emits photons into previously unoccupied modes. The atom thus fluoresces. Our goal is to study the quantum-optical properties of this fluorescense which plays a fundamental role in our understanding of atom-photon interactions.

Consider the following geometry:

Photon scattering



The Hamiltonian for the system in the RWA

$$\hat{H} = \frac{\hbar\omega_0}{2}\hat{\sigma}_z + \sum_{\vec{k},\mu}\hbar\omega_k \hat{a}^\dagger_{\vec{k}\mu}\hat{a}_{\vec{k}\mu} + \sum_{\vec{k}\mu}\hbar\left(g_{\vec{k}\mu}\hat{\sigma}_+\hat{a}_{\vec{k}\mu} + g^*_{\vec{k}\mu}\hat{a}^\dagger_{\vec{k}\mu}\hat{\sigma}_-\right)$$

We take as the initial state, $|\Psi\rangle = |\psi\rangle_A \otimes |\alpha_L e^{-i\omega_L t}\rangle_{\vec{k}_L} \otimes |0\rangle_{\vec{k}\neq\vec{k}_L}$

The laser mode is initially in a coherent state - all others modes in vacuum.

We can substantially simplify the analysis to take into account the coherent interaction with the laser by making a unitary transformation, displacing the laser mode's state back to the vacuum, while simultaneously displacing the operators.

"Mollow picture": $|\Psi\rangle \Rightarrow \hat{D}^\dagger_{\vec{k}_L}(\alpha_L(t))\,|\Psi\rangle$, $\quad\hat{O} \Rightarrow \hat{D}^\dagger_{\vec{k}_L}(\alpha_L(t))\,\hat{O}\,\hat{D}_{\vec{k}_L}(\alpha_L(t))$

$$\hat{D}_{\vec{k}_L}(\alpha_L(t)) = \exp\left\{\alpha_L(t)\hat{a}^\dagger_{\vec{k}_L} - \alpha^*_L(t)\hat{a}_{\vec{k}_L}\right\}$$

In this picture: $|\Psi\rangle \Rightarrow |\psi\rangle_{Atom} \otimes |0\rangle_{all\ modes}$

$$\hat{a}_{\vec{k}} \Rightarrow \hat{a}_{\vec{k}} + \alpha_L e^{-i\omega_L t}$$

Note: $\hbar g_{\vec{k}_L} \alpha_{k_L} = \vec{d}_{eg}\cdot\vec{\epsilon}_L \sqrt{\dfrac{2\pi\hbar\omega_k |\alpha_L|^2}{V}} = \dfrac{\vec{d}_{eg}\cdot\vec{\epsilon}_L}{2}\sqrt{\dfrac{8\pi\ \hbar\omega_L \bar{n}_L}{V}} = \dfrac{\vec{d}_{eg}\cdot\vec{E}_L}{2} = \dfrac{\hbar\Omega}{2}$ ← Rabi freq.

Thus in this frame: $\hat{H} = \underbrace{\hat{H}_A}_{\text{atom}} + \underbrace{\hat{H}_{AL}}_{\text{atom-laser}} + \underbrace{\hat{H}_V}_{\text{vacuum}} + \underbrace{\hat{H}_{AV}}_{\text{atom-vacuum}}$

$$\hat{H}_A + \hat{H}_{AL} = \frac{\hbar\omega_0}{2}\hat{\sigma}_z + \frac{\hbar\Omega}{2}\left(\hat{\sigma}_+ e^{-i\omega_L t} + \hat{\sigma}_- e^{i\omega_L t}\right) \quad \text{(Semi-classical Hamiltonian)}$$

$$\hat{H}_V + \hat{H}_{AV} = \sum_{\vec{k},\mu}{}' \hbar\omega_k \hat{a}^\dagger_{\vec{k}\mu}\hat{a}_{\vec{k}\mu} + \sum_{\vec{k}\mu}\hbar\left(g_{\vec{k}\mu}\hat{\sigma}_+\hat{a}_{\vec{k}\mu} + g^*_{\vec{k}\mu}\hat{a}^\dagger_{\vec{k}\mu}\hat{\sigma}_-\right)$$

We recover the semi-classical Hamiltonian studied at the begin of the semester plus the coupling of the atom to the vacuum. We are able to achieve this because the quantum fluctuations of the coherent state are the same as those for the vacuum.

The formal solution for the Heisenberg equation of motion of the field is as previously studied:

$$\hat{a}_{\vec{k}\mu}(t) = \hat{a}_{\vec{k}\mu}(0)e^{-i\omega_k t} + \int_0^t dt' e^{-i\omega_k(t-t')}\sqrt{\frac{2\pi\hbar\omega_k}{V}}\,\vec{d}_{eg}\cdot\vec{\epsilon}^{\,*}_{\vec{k}\mu}\,\hat{\sigma}_-(t')$$

$$\Rightarrow \hat{E}^{(+)}(\vec{r},t) = \hat{E}^{(+)}_{vac}(\vec{r},t) + \hat{E}^{(+)}_{source}(\vec{r},t)$$

$$\vec{E}^{(+)}_{vac}(\vec{r},t) = \sum_{\vec{k}\mu}{}'\sqrt{\frac{2\pi\hbar\omega_k}{V}}\,\hat{a}_{\vec{k}\mu}(0)\,\vec{\epsilon}_{\vec{k}\mu}\,e^{i(\vec{k}\cdot\vec{r}-\omega_k t)}$$

$$\vec{E}_{source}(\vec{r},t) = \underbrace{-\frac{\omega_0^2}{c^2}\frac{(\vec{d}_{eg})_\perp}{r}}_{\equiv\,\eta}\hat{\sigma}_-\left(t-\frac{r}{c}\right) \quad \text{(dipole radiation)}$$

In contrast to our previous study of spontaneous emission, now the atom is driven by the coherent laser field ⇒ Dynamics of $\hat{\sigma}_-(t)$

Elastic (coherent) vs. Inelastic (incoherent) Scattering

Let us first study the intensity of the scattered light @ the position of a detector in the far field:

$$I(\vec{r},t) = \langle \hat{E}^{(-)}_{(\vec{r},t)} \hat{E}^{(+)}(\vec{r},t) \rangle = G^{(1)}(\vec{r},t;\ \vec{r},t) = \eta^2 \langle \hat{\sigma}_+(t-\tfrac{r}{c})\, \hat{\sigma}_-(t-\tfrac{r}{c}) \rangle$$

$$= \eta^2 \langle (|e\rangle\langle g|)(|g\rangle\langle e|)(t-\tfrac{r}{c}) \rangle = \eta^2 P_e(t-\tfrac{r}{c}) \leftarrow \text{Probability to be in the excited state}$$

This makes sense physically: If we detect a photon at the detector at time t, then at the retarded time, the atom was in the excited state (we take the spontaneous jump to be instantaneous.

We can decompose the dipole operator in terms of its mean value + fluctuations

$$\hat{\sigma}_-(t) = \langle \hat{\sigma}_-(t) \rangle + \delta\hat{\sigma}_-(t)$$

, where $\langle \delta\hat{\sigma}_+ \rangle = 0$ in state

Note $\langle \hat{\sigma}_-(t) \rangle = \underbrace{\mathrm{Tr}(\hat{\sigma}_-(t)\, \hat{\rho})}_{\text{Heisenberg picture}} = \underbrace{\mathrm{Tr}(\hat{\sigma}_-^{RF}(t)\, e^{-i\omega_L t}\, \hat{\rho}_{RF}(t))}_{\text{Rotating frame}} = e^{-i\omega_L t} \langle \hat{\sigma}_- \rangle_{RF}$

$\Rightarrow$ $\langle \hat{\sigma}_-(t) \rangle$ is the "coherent radiation" driven by the laser

$\delta\hat{\sigma}_+$ represents the quantum fluctuations in the field due to vacuum fluctuations "incoherent"

$$\Rightarrow I(\vec{r},t) = \underbrace{\eta^2 |\langle \hat{\sigma}_+(t-\tfrac{r}{c}) \rangle_{RF}|^2}_{I_{coh}} + \underbrace{\eta^2 \langle \delta\hat{\sigma}_+(t-\tfrac{r}{c})\ \delta\hat{\sigma}_-(t-\tfrac{r}{c}) \rangle_{RF}}_{I_{incoh}}$$

Consider now the intensity observed after atom response reaches steady-state.
In steady-state, the expectation values are

$$\langle \hat{\sigma}_+ \hat{\sigma}_- \rangle_{s.s.} = P_e = \frac{1 + w^{s.s.}}{2} = \frac{s}{2(1+s)}$$

$$|\langle \hat{\sigma}_+ \rangle|^2 = \left| \frac{u^{s.s.} + i v^{s.s.}}{2} \right|^2 = \frac{s}{2(1+s)}$$

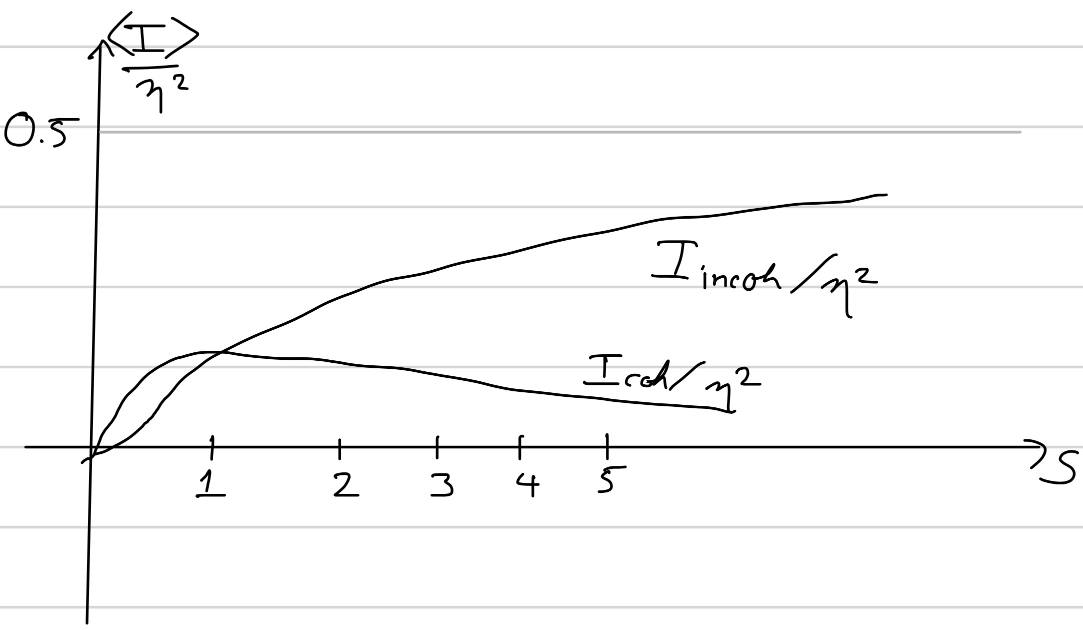
$$s = \frac{\Omega^2/2}{\Delta^2 + \frac{\Gamma^2}{4}} \quad \text{(saturation parameter)}$$

$$\Rightarrow \frac{1}{\eta^2} I_{coh} = |\langle\hat{\sigma}_+\rangle_{s.s.}|^2 = \frac{1}{2}\frac{s}{(1+s)^2}$$

$$\frac{1}{\eta^2} I_{incoh} = \frac{1}{\eta}\left(I - I_{coh}\right) = \langle\hat{\sigma}_+\hat{\sigma}_-\rangle - |\langle\hat{\sigma}_+\rangle|^2 = \frac{1}{2}\frac{s^2}{(1+s)^2}$$

Note: For $s \ll 1$ $\frac{I_{coh}}{\eta^2} \approx \frac{s}{2} \gg \frac{1}{\eta^2} I_{incoh} \approx \frac{s^2}{2}$

For low saturation, coherent scattering dominates of incoherent scattering



For $s \ll 1$ the radiated intensity $\propto s \propto I_L \Rightarrow$ scattering is coherent and we can define a scattering cross section. As $s \to \infty$ $I_{coh} \to 0$ is the transition is saturated (mean dipole $\to 0$). In this regime the radiated field is incoherent arising primarily from vacuum induced jumps.

Spectrum of fluorescence

To calculate the spectrum of the fluorescence, we need to calculate the Fourier transform of two-time correlation function

$$S(\omega) = \eta^2 \int_{-\infty}^{\infty} d\tau \, G^{(1)}(\tau) \, e^{-i\omega\tau}$$

$$G^{(1)}(\tau) = \langle \hat{E}^{(-)}(\bar{r}, t+\tau) \, \hat{E}^{(+)}(\bar{r}, t) \rangle = \langle \hat{\sigma}_+(t - \tfrac{r}{c} + \tau) \, \hat{\sigma}_-(t - \tfrac{r}{c}) \rangle$$

$$= \langle \hat{\sigma}_+^{RF}(t - \tfrac{r}{c} + \tau) \, \hat{\sigma}_+^{RF}(t - \tfrac{r}{c}) \rangle_{RF} \, e^{i\omega_L \tau} \quad (\text{in } RF)$$

$\Rightarrow S(\omega) = S_{coh}(\omega) + S_{incoh}(\omega)$

$$S_{coh}(\omega) = \frac{\eta^2}{2\pi}\int_{-\infty}^{\infty} d\tau\, e^{i(\omega_L-\omega)\tau}\, |\langle\hat{\sigma}_+\rangle_{s.s.}|^2$$

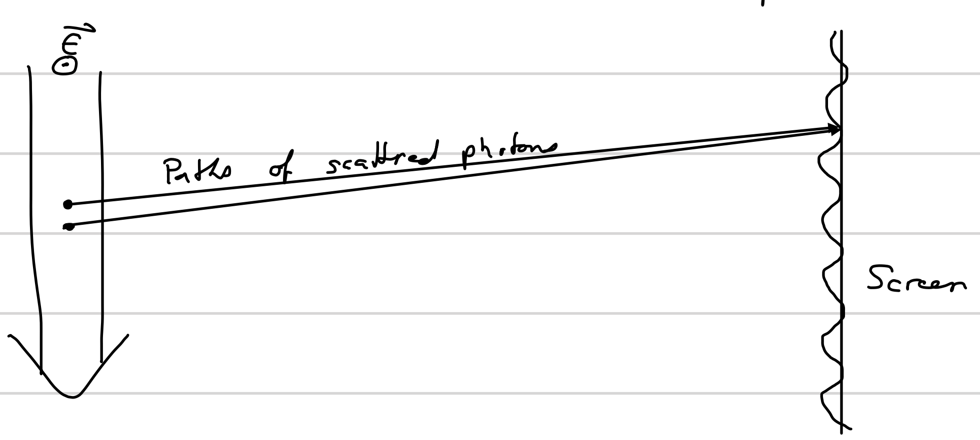
$$S_{incoh}(\omega) = \frac{\eta^2}{2\pi}\int_{-\infty}^{\infty} d\tau\, e^{i(\omega_L-\omega)\tau}\, \langle\delta\sigma_+(\tau)\,\delta\sigma_-(0)\rangle_{s.s.}$$

[Note: in s.s., statistics are "stationary". Everything is in the rotating frame]

$\Rightarrow S_{coh}(\omega) = I_{coh}\,\delta(\omega_L-\omega)$: Coherent scattering is elastic.
The coherent radiation is at the same freq. as the laser freq.

The elastic scattering is associated with the linear response of the atomic dipole $\langle\hat{d}_{ind}\rangle = \alpha\,\vec{E}_L(\vec{R}_{atom},t)$. In this regime the atom radiates as a dipole antenna, with a phase induced by the laser field. These classically scattered photons can interfere with one anoth

Gedanken experiment: Laser beam detuned from resonance with $s \ll 1$ excited trapped atoms that flouresce. The scattered photons are detected on a far screen



The result will be an "double slit" interference pattern. We have no information about which atom scatters the photon, but it is elastic scattering when $s \ll 1$. Because it is coherent, these two paths interfere. Note: Scattered photon is not definite number state

$$|\psi\rangle_{scat} \approx \sum_{\vec{k}}\left(|0\rangle + 2\pi i\, g^*_{\vec{k}\mu}\, e^{i(\vec{k}-\vec{k}_L)\cdot\vec{R}_{atom}}\, \frac{\Omega(\vec{R})/2}{i\Delta-\frac{\Gamma}{2}}\,\delta(\omega_k-\omega_L)\right) \quad (\text{for homework})$$

In contrast, the incoherent radiation is not monochromatic, and will have other frequency components due to fluctuating dipole.

We seek $S_{incoh}(\omega) = \frac{\eta^2}{2\pi}\int_{-\infty}^{\infty} d\tau \; e^{i(\omega_L-\omega)\tau} \langle \delta\hat{\sigma}_+(\tau)\, \delta\hat{\sigma}_-(0)\rangle_{RF}$

We need to solve for $\langle \delta\hat{\sigma}_+(\tau)\, \delta\hat{\sigma}_-(0)\rangle_{RF}$

<u>Quantum Regression theorem</u>

To solve for $\langle \delta\hat{\sigma}_+(\tau)\,\delta\hat{\sigma}_-(0)\rangle$ one can use the optical Bloch equations in the Heisenberg picture. Recall

$$\frac{d}{dt}\langle\hat{\sigma}_+\rangle = \left(-i\Delta - \frac{\Gamma}{2}\right)\langle\hat{\sigma}_+\rangle - i\frac{\Omega}{2}\langle\hat{\sigma}_z\rangle$$
$$\frac{d}{dt}\langle\hat{\sigma}_z\rangle = -\Gamma\left(\frac{1+\langle\hat{\sigma}_z\rangle}{2}\right) - i\frac{\Omega}{2}\left[\langle\hat{\sigma}_+\rangle - \langle\hat{\sigma}_-\rangle\right]$$

This has a linear form: $\frac{d}{dt}\langle\hat{\sigma}_i\rangle = \sum_{i,j} B_{ij}\langle\hat{\sigma}_j\rangle$

We will show next semester that in the <u>Markoff approximation</u>, the evolution of two-time correlation functions "regress to" one-time correlations when $\tau \geq 0$

$$\boxed{\frac{d}{d\tau}\langle\delta\hat{\sigma}_i(\tau)\,\delta\hat{\sigma}_-(0)\rangle = \sum_j B_{ij}\langle\delta\hat{\sigma}_i(\tau)\,\delta\hat{\sigma}_j(0)\rangle}$$ Quantum regression theorem.

The frequency components of $\langle\delta\sigma_+(\tau)\,\delta\sigma_-(0)\rangle$ thus depend on the eigenvalues of B_{ij}

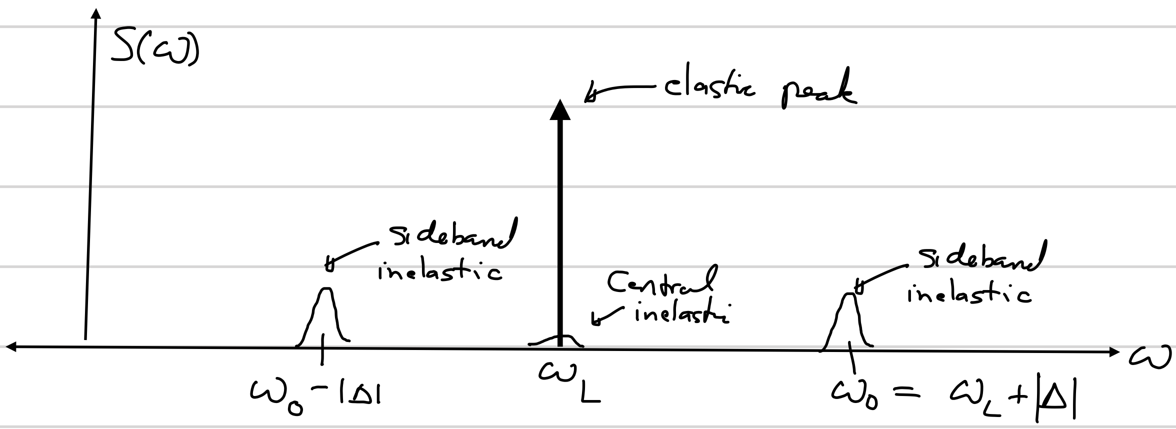
Limiting Cases:

<u>Far off resonance, low saturation</u>: $\Delta \gg \Omega, \Gamma$ $\quad S \ll 1$ (mostly elastic)

Eigenvalues: $r_1 = -2\Gamma \quad r_2 = -i(\omega_L - \omega_0) - \Gamma, \quad r_3 = -i(\omega_0 - \omega_L) - \Gamma$

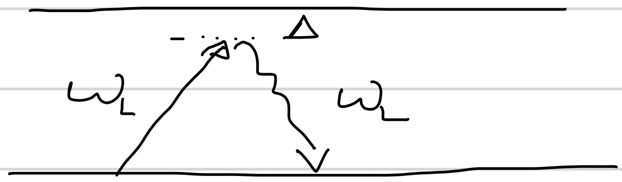
$$\Rightarrow S(\omega) = \underbrace{\int d\tau\, S_1(\tau)\, e^{(i\omega_L - 2\Gamma)\tau}}_{\text{line centered at } \omega_L \text{ of with } 2\Gamma} + \underbrace{\int d\tau\, S_2\, e^{(i\omega_0 - \Gamma)\tau}}_{\text{line centered at } \omega_L+\Delta=\omega_0,\ \text{width } \Gamma} + \underbrace{\int d\tau\, S_3\, e^{(i2\omega_L - \omega_0 - \Gamma)\tau}}_{\text{line center at } \omega_L-\Delta = 2\omega_L-\omega_0 \text{ + width } \Gamma}$$

Fluorescence spectrum: Off resonance, low saturation

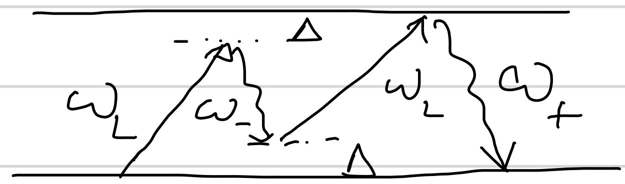


For this low intensity regime, we can understand the spectrum in perturbation theory as dominant one-photon processes, and higher order two-photon process

Elastic scattering: One photon:



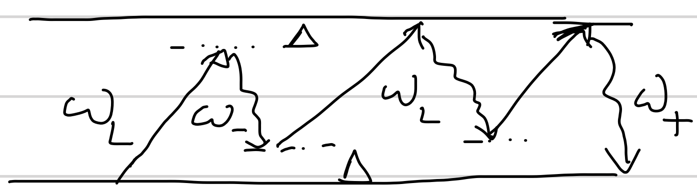
Inelastic sidebands: Two-photons



$$\omega_- = \omega_L - |\Delta|$$
$$\omega_0 = \omega_L + |\Delta| = \omega_0$$

The photons scattered into sidebands come in pairs. The this nonlinear optical process is a first example of the way in which nonclassical light is generated (photon correlations).

Inelastic central peak and sidebands ⇒ Three photons

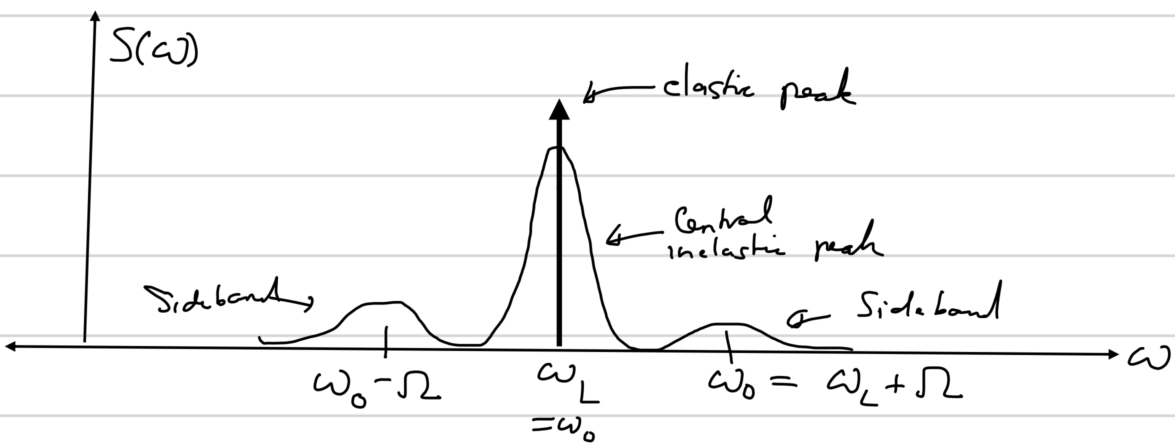


The central peak will be highly suppressed w.r.t. the sidebands in the low saturation regime.

Note: There is always a temporal correlation in the emission of the inelastic photons: ω_- always comes first, followed by ω_L or ω_+.

High saturation regime (on resonance): $\Delta = 0$ $\quad \Omega \gg \Gamma$

Eigenvalues $r_0 = -\Gamma$ $\quad r_\pm = \pm i\Omega - \frac{3}{2}\Gamma$



This inelastic spectrum is known as the Mollow triplet following B.R. Mollow who first calculated it (B.R. Mollow, Phys. Rev. 188, 1969 (1969)). In the high saturation regime, we cannot use perturbation theory, making the interpretation in terms of elementary photon absorption and emission processes difficult. One way to understand the spectrum is via amplitude modulation:

$$\bar{E}_{rad} = \mathrm{Re}\left(\langle \vec{d}(t)\rangle e^{-i\omega_L t}\right)$$

$$\langle \vec{d}(t)\rangle \propto u(t) + i\, v(t)$$ ← oscillate at Rabi frequency

$\Rightarrow$ Sidebands at $\omega_L \pm \Omega$

The emergence of the Mollow-spectrum is a signature of damped Rabi oscillations.

Dressed-State Picture

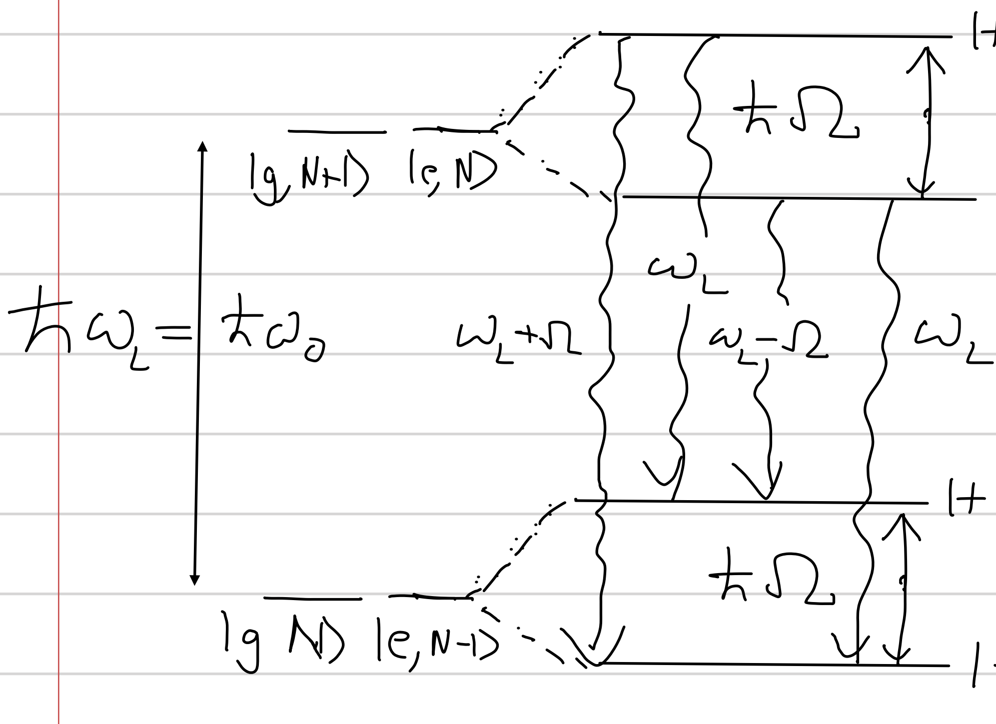
The strong coupling of the laser field to atoms in the high saturation regime implies perturbation theory is not accurate and we cannot easily interpret the Mollow triplet in terms of elementary processes. We can however obtain an intuitive (and ultimately quantitative picture) by going to a dressed basis of atoms and the photons of the laser field. This dressed-state picture was championed by Cohen-Tannoudji, who enabled deeper insight into resonance fluorescence.

Instead of making the Mollow transformation, consider diagonalization of Atom + Quantized Laser Mode:

$$\hat{H}_A + \hat{H}_L + \hat{H}_{AL} = \frac{\hbar\omega_0}{2}\hat{\sigma}_z + \hbar\omega_L \hat{a}_L^{\dagger}\hat{a}_L + \hbar g_L(\hat{a}_L\hat{\sigma}_+ + \hat{\sigma}_-\hat{a}_L^{\dagger})$$

This is the Jaynes-Cummings Hamiltonian we studied in Cavity QED, except now in the artificial "quantization volume" $V \to \infty$. We take # of photons in laser mode $N \to \infty$, so $\Delta N \ll N$ and $N/V \to$ constant $\frac{N}{V} 8\pi\hbar\omega_L = E_L^2$

We thus obtain the Jaynes-Cumming type ladder



$$|+(N+1)\rangle = \frac{|g,N+1\rangle + |e,N\rangle}{\sqrt{2}}$$

$$|-(N+1)\rangle = \frac{|g,N+1\rangle - |e,N\rangle}{\sqrt{2}}$$

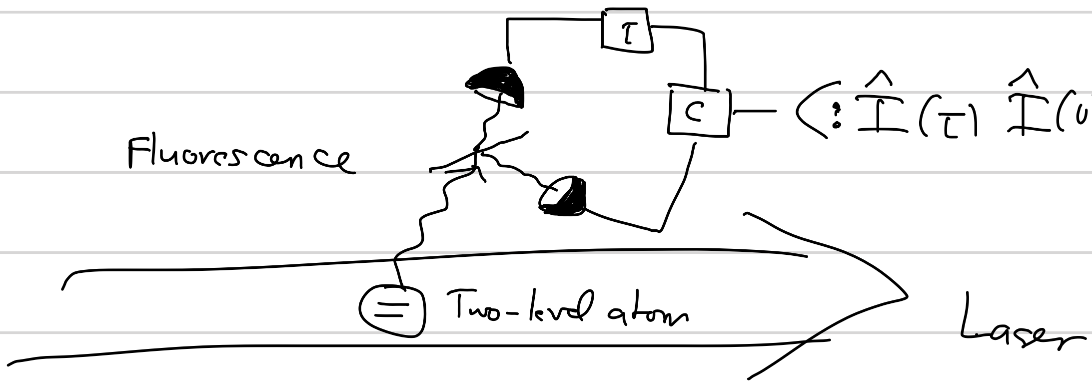
$$|+(N)\rangle = \frac{|g,N\rangle + |e,N-1\rangle}{\sqrt{2}}$$

$$|-(N)\rangle = \frac{|g,N\rangle - |e,N-1\rangle}{\sqrt{2}}$$

We see that there are radiative processes that take dressed states in one manifold with N+1-photons to a lower manifold with N photons. The <u>dressed state splitting</u> leads to side bands.

<u>Second-Order Coherence</u>

Let us now consider the $G^{(2)}$ correlation function which characterizes the photon statistics and classical vs. nonclassical features of the fluorescence. Let us study the temporal coherence in a Hanbury-Brown & Twiss type experiment



$$\langle :\hat{I}(\tau)\,\hat{I}(0): \rangle = G^{(2)}(\tau)$$

The fluorescence is passed through a beam splitter, and we look for coincidence counts between two detection events separated by time τ.

$$G^{(2)}(\tau) = \langle \hat{E}^{(-)}(0)\,\hat{E}^{(-)}(\tau)\,\hat{E}^{(+)}(\tau)\,\hat{E}^{(+)}(0) \rangle = \langle \hat{E}^{(-)}_{Source}(0)\,\hat{E}^{(-)}_{Source}(\tau)\,\hat{E}^{(+)}_{Source}(\tau)\,\hat{E}^{(+)}_{Source}(0) \rangle$$

Note: While $[\hat{E}^{(+)}(t_1), \hat{E}^{(+)}(t_2)] = 0$, $[\hat{E}^{(+)}_{Source}(t_1), \hat{E}^{(+)}_{Source}(t_2)] \neq 0$ so operator ordering is important so that $G^{(2)}(\tau)$ is real. Moreover, generally $[\hat{E}^{(+)}_{vac}(\tau), \hat{E}^{(-)}_{Source}(0)] = 0$ only if $\tau > 0$ by causality (see Mollow)

$$\Rightarrow \quad G^{(2)}(\tau) = \eta^2 \langle \hat{\sigma}_+(0)\,\hat{\sigma}_+(\tau)\,\hat{\sigma}_-(\tau)\,\hat{\sigma}_-(0) \rangle$$

$$= \eta^2\, \mathrm{Tr}\left(|e\rangle\langle g|(0)\; |e\rangle\langle e|(\tau)\; |g\rangle\langle e|(0)\; \hat{\rho}_{S.S.} \right)$$

$$= \eta^2\, \mathrm{Tr}\left(|e\rangle\langle e|(\tau)\; |g\rangle\langle e|\, \hat{\rho}_{SS}\, |e\rangle\langle g|(0) \right)$$

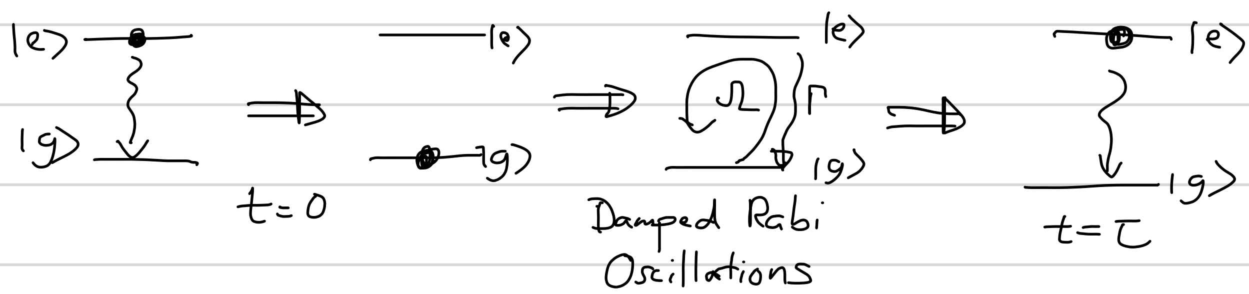
$$= \eta^2\, \mathrm{Tr}\left(|e\rangle\langle e|(\tau)\; |g\rangle\langle g|(0) \right)\, P_e(0)$$

Aside: $\mathrm{Tr}(|e\rangle\langle e|(\tau)\ |g\rangle\langle g|(0)) = |\langle e;\tau|g;0\rangle|^2 = P(e;\tau|g,0)$

Conditional probability that the atom is in e @ time τ given it was in g @ $t=0$

Thus $\boxed{G^{(2)}(\tau) = \eta^2\ P(e;\tau|g,0)\ P_e(0)}$

We can understand the physical meaning of this result as follows. $G^{(2)}(\tau)$ measures the probability of detecting a photon at time $t+\tau$ given that one was detected at time t (for stationary statistics, this probability is independent of t, which we call $t=0$ for convenience). At $t=0$ (including retardation) a photon was emitted $\Rightarrow$ at $t=0$ the atom was in $|e\rangle$ and then jumped to $|g\rangle$. Thus the probability of seeing the first photon is determined by the probability that the atom was in $|e\rangle$ at time 0. In the Markoff approximation, the jump for $|e\rangle \to |g\rangle$ is instantaneous. Thus measuring the first photon immediately projects the atom into $|g\rangle$. The probability to detect the second photon at the later time τ then depends on the probability of reëxciting the atom so that at time τ it is in $|e\rangle$ and then makes a second jump.



From this description is it clear that we expect the scattering photons to be antibunched. After the detection of the first photon, the atom is immediately projected into the ground state. It takes a finite time for the atom to be reëxcited, and thus, the second photon will arrive later. A single atom cannot simultaneously emit two photons on a given two-level resonance. Note, the $G^{(2)}(\tau)$ correlation function does not measure the probability that the next photon was emitted at time τ given one emitted at $t=0$, only that there is an emission at τ. Thus between 0 and τ the atom undergoes damped Rabi oscillations.

We have from the optical Bloch equations

$$P_e(0) = P_e^{s.s.} = \frac{s}{2(1+s)}$$ (the initial time is after atom has reach steady state)

$$P_e(e;\tau|g;0) = \langle g;0|\hat{P}_e(\tau)|g;0\rangle = \frac{1+\langle g;0|\hat{\sigma}_z(\tau)|g;0\rangle}{2}$$

Torrey solution on resonance: $$\langle g;0|\hat{\sigma}_z(\tau)|g;0\rangle = -1 + \frac{\Omega^2}{\Omega^2+\frac{\Gamma^2}{2}}\left[1 - e^{-\frac{3}{4}\Gamma t}\left(\cos\tilde{\Omega}t + \frac{3}{4}\frac{\Gamma}{\tilde{\Omega}}\sin\tilde{\Omega}t\right)\right]$$

where $\tilde{\Omega} = \sqrt{\Omega^2 - \frac{\Gamma^2}{4}}$

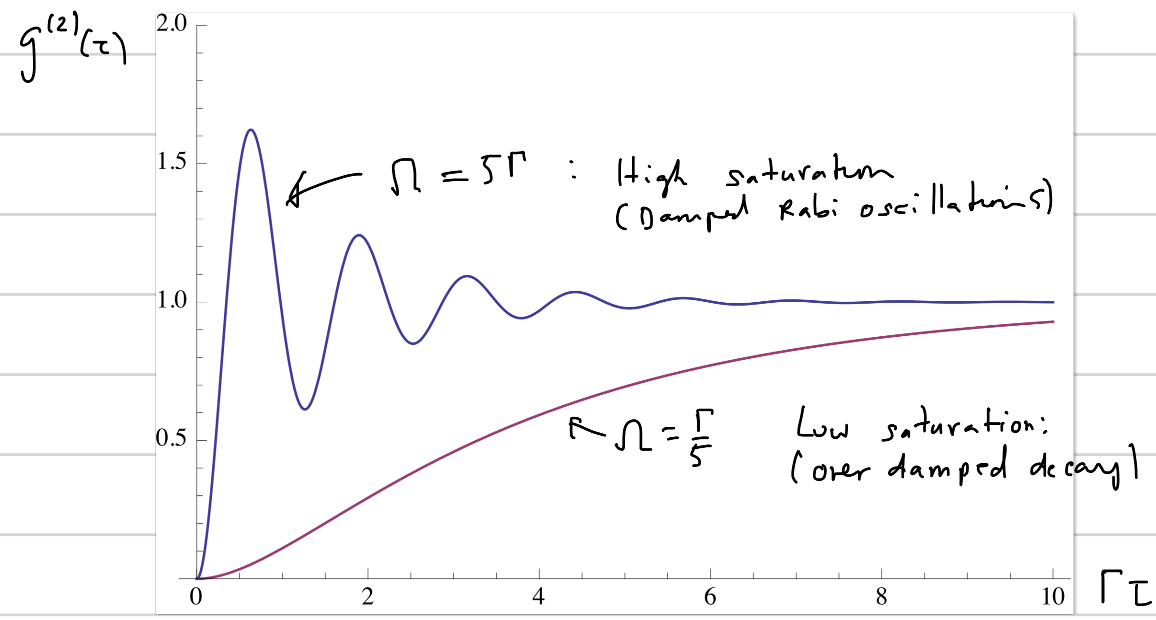
We obtain the normalized correlation function

$$g^{(2)}(\tau) = \frac{G^{(2)}(\tau)}{[G^{(1)}(\tau)]^2} = \frac{G^{(2)}(\tau)}{\langle I\rangle^2} = \frac{G^{(2)}(\tau)}{[\eta P_{ee}^{s.s.}]^2} = P(e;\tau|g,0)$$

On resonance: $$P_e^{s.s.} = \frac{\Omega^2/4}{\Omega^2+\frac{\Gamma^2}{2}}$$

$$\Rightarrow \boxed{g^{(2)}(\tau) = 1 - e^{-\frac{3}{4}\Gamma\tau}\left(\cos(\tilde{\Omega}\tau) + \frac{3\Gamma}{4\tilde{\Omega}}\sin(\tilde{\Omega}\tau)\right)}$$

$\boxed{\text{Note } g^{(2)}(0_+) = 0 \Rightarrow \text{Photon antibunching}}$



Photon antibunching