Physics 566 - Quantum Optics I

Lecture 16: Quantum Optical Coherence

The core subject of quantum optics is coherence – the capacity for system to exhibit interference. In particular we are interested in quantum coherence – i.e., interference between alternative quantum processes – associated with electromagnetic fields. For the majority of the course so far, we have focussed on atomic coherence, i.e., coherent superposition of atomic energy levels for which the atomic reponse to electromagnetic fields is nonclassical, e.g. Rabi oscillations and electromagnetically induced transparency.

We now want to turn our attention to the electromagnetic field itself. This is a subtle business. The electromagnetic fields are described classically as waves, so there is a sense in which coherence in electromagnetism is a classical phenomenon. But electromagnetic fields are also described by particles, so there is a sense in which coherence is a quantum phenomenon associated with the interference of paths the particle takes. We saw this early in the course. The interference in a Mach-Zender interferometer could alternatively be described by interfering classical waves or by interference of probability amplitudes associated with two indistinguishable paths a photon can take on its way to a detector. So, although there is a quantum explanation underlying the observed interference fringes, the phenomenon is "essentially classical" in nature, in that the classical theory of electromagnetic waves gives the proper prediction of the observations. Once we include the semiclassical description of photon detection (quantum absorbers), we need not quantize the field to describe Mach-Zender-type interference.

Our goal, thus, is to study quantum optical coherence, and to understand the conditions under which this is irreducibly quantum mechanical in nature, and when the classical theory can explain the phenomenon. This distinction allows us to distinguish classical light vs. nonclassical light.

Review: Classical Statistical Optics

To distinguish "classical light" from "nonclassical light" we first review the classical theory, studied at the beginning of the semester. In particular it is important to understand classical statistical optics, whereby the complex wave amplitude $\tilde{E}(\vec{r},t)$ is a random variable due to our incomplete knowledge of the source that produced the field.

We write the field as decomposition into modes

$$\mathcal{E}(\vec{r},t) = \sum_{\vec{k}} C_{\vec{k}} \frac{e^{i(\vec{k}\cdot\vec{r} - \omega_k t)}}{\sqrt{V}}$$

(for simplicity, we take the field to be polarized)

The classical "state of field" is determined by probability distribution we assign to the mode amplitudes: $P(\{C_k\}, t)$. We typically make the following assumptions:

(1) The statistics are "stationary," i.e. $P(\{C_k\})$ is independent of t.

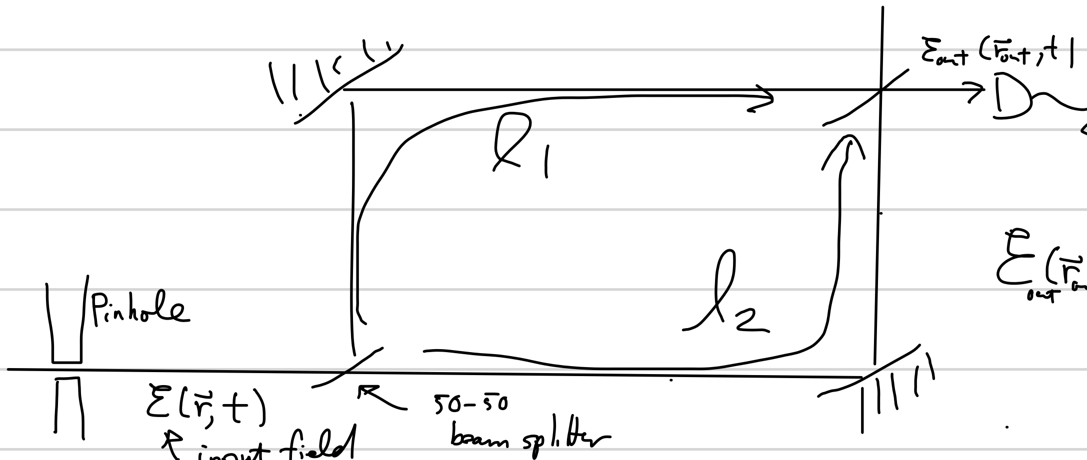
(This doesn't mean that nothing changes as function of time, just that the statistics are constant)

(2) The dynamics are "ergodic" $\Rightarrow$ Sampling the field at different times is equivalent to sampling from the probability distribution $P(\{C_k\})$ $\Rightarrow$ expectation values are equivalent to time averages.

$$\left\langle \tilde{\mathcal{F}}(\mathcal{E}(\vec{r},t)) \right\rangle = \int d^2\{C_k\}\, P(\{C_k\})\, \mathcal{F}(\mathcal{E}(\vec{r},t)) \equiv \lim_{T\to\infty} \frac{1}{T}\int_0^T dt\, \mathcal{F}(\mathcal{E}(\vec{r},t)) = \overline{\mathcal{F}(\mathcal{E}(\vec{r},t))}$$

The ergodic assumption is a good approximation for natural light sources.

Consider now the Mach-Zender interferometer:



$$\langle I_{out}\rangle = \langle |\mathcal{E}_{out}(\vec{r}_{out},t)|^2\rangle$$

$$\mathcal{E}_{out}(\vec{r}_{out},t) = \frac{1}{2}\left(\mathcal{E}(\vec{r},t_1) + \mathcal{E}(\vec{r},t_2)\right)$$

(up to overall phase)

$$t_1 = t - \frac{l_1}{c}, \quad t_2 = t - \frac{l_2}{c}$$

$$\Rightarrow \langle I_{out}\rangle = \frac{1}{4}\left[\langle|\mathcal{E}(t_1)|^2\rangle + \langle|\mathcal{E}(t_2)|^2\rangle + 2\,\mathrm{Re}\,\underbrace{\langle \mathcal{E}^*(t_1)\,\mathcal{E}(t_2)\rangle}_{\text{temporal correlation function}}\right]$$

(Dropping position dependence because all $\mathcal{E}$ @ same $\vec{r}$)

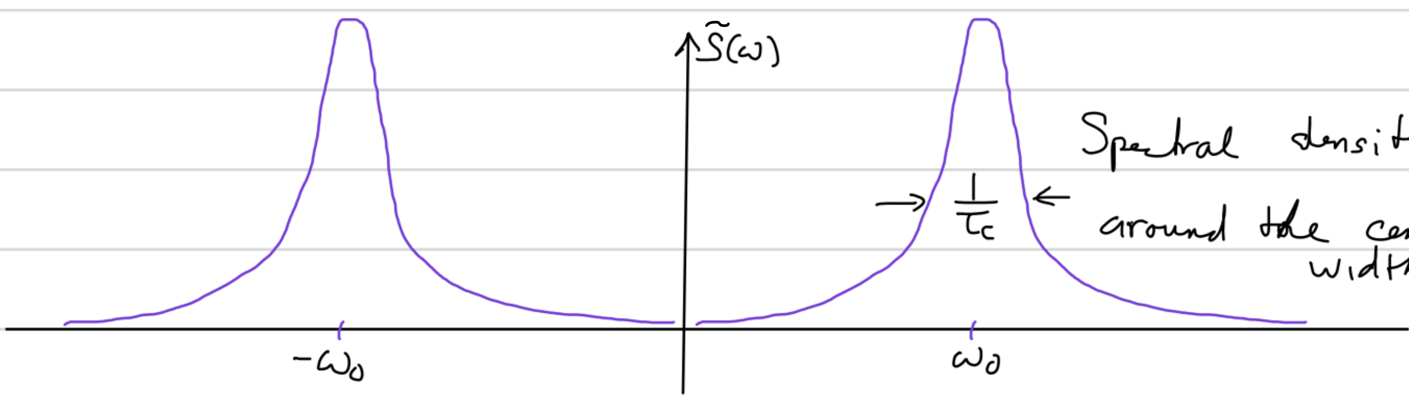
For stationary statistics $\langle|\mathcal{E}(t_1)|^2\rangle = \langle|\mathcal{E}(t_2)|^2\rangle = \langle I_{in}\rangle$, $\langle \mathcal{E}^*(t_1)\,\mathcal{E}(t_2)\rangle = \langle \mathcal{E}^*(\tau)\,\mathcal{E}(0)\rangle$, $\tau = t_1 - t_2$

$$\langle I_{out}\rangle = \frac{\langle I_{in}\rangle}{2}\left[1 + \frac{\mathrm{Re}\left(\langle \mathcal{E}^*(\tau)\,\mathcal{E}(0)\rangle\right)}{\langle I_{in}\rangle}\right]$$

According to the Weiner-Khinchine theorem, studied earlier

$$\langle \mathcal{E}^*(\tau)\,\mathcal{E}(0)\rangle = \int_0^\infty \frac{d\omega}{2\pi}\,\tilde{S}(\omega)\,e^{-i\omega t}$$ = Positive frequency component of the Fourier transform of the spectral density $\langle \tilde{\mathcal{E}}^*(\omega)\,\tilde{\mathcal{E}}(\omega')\rangle = \tilde{S}(\omega)\,\delta(\omega-\omega')$

Example: Collision broadened "natural light" $\tilde{S}(\omega) = \frac{\langle I_{in}\rangle}{2}\left[\frac{\frac{1}{\tau_c}}{(\omega-\omega_0)^2 + \left(\frac{1}{\tau_c}\right)^2} + \frac{\frac{1}{\tau_c}}{(\omega+\omega_0)^2 + \left(\frac{1}{\tau_c}\right)^2}\right]$



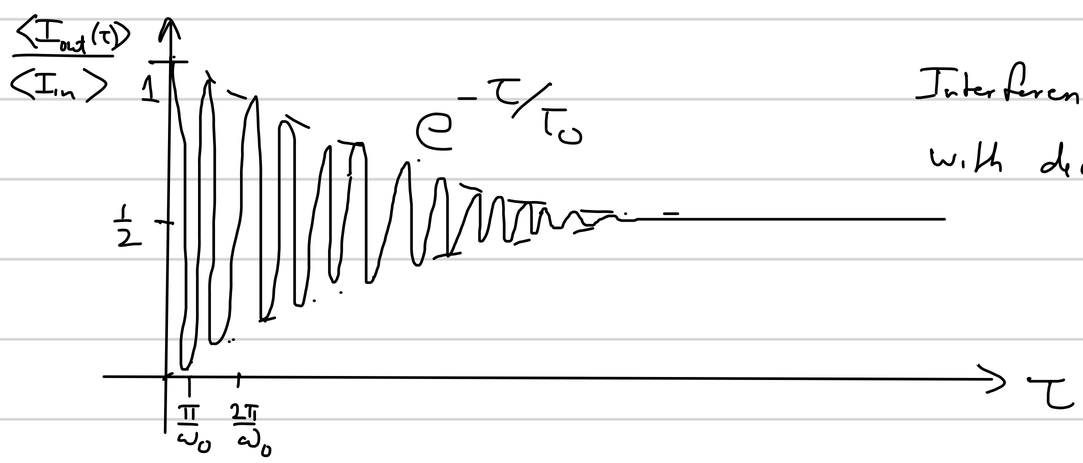
Spectral density is Lorentzian around the central freq., width $\cong \frac{1}{\tau_c}$

$$\tilde{S}(\omega) = \tilde{\gamma}(\omega) \circledast \langle I_{in}\rangle \frac{\delta(\omega-\omega_0) + \delta(\omega+\omega_0)}{2}, \qquad \gamma(\omega) = \frac{\frac{1}{\tau_c}}{\omega^2 + \left(\frac{1}{\tau_c}\right)^2}$$

(Convolution)

$$\Rightarrow \langle \mathcal{E}^*(\tau)\,\mathcal{E}(0)\rangle = \gamma(\tau)\langle I_{in}\rangle e^{-i\omega_0\tau}, \qquad \gamma(\tau) = \int \frac{d\omega}{2\pi}\,\gamma(\omega)\,e^{-i\omega\tau} = \langle I_{in}\rangle e^{-\tau/\tau_c}$$

$$\Rightarrow \langle I_{out}(\tau)\rangle = \frac{\langle I_{in}\rangle}{2}\left(1 + \gamma(\tau)\cos\omega_0\tau\right) \qquad \tau = \frac{l_1 - l_2}{c}$$



Interference fringes with decaying visibility.

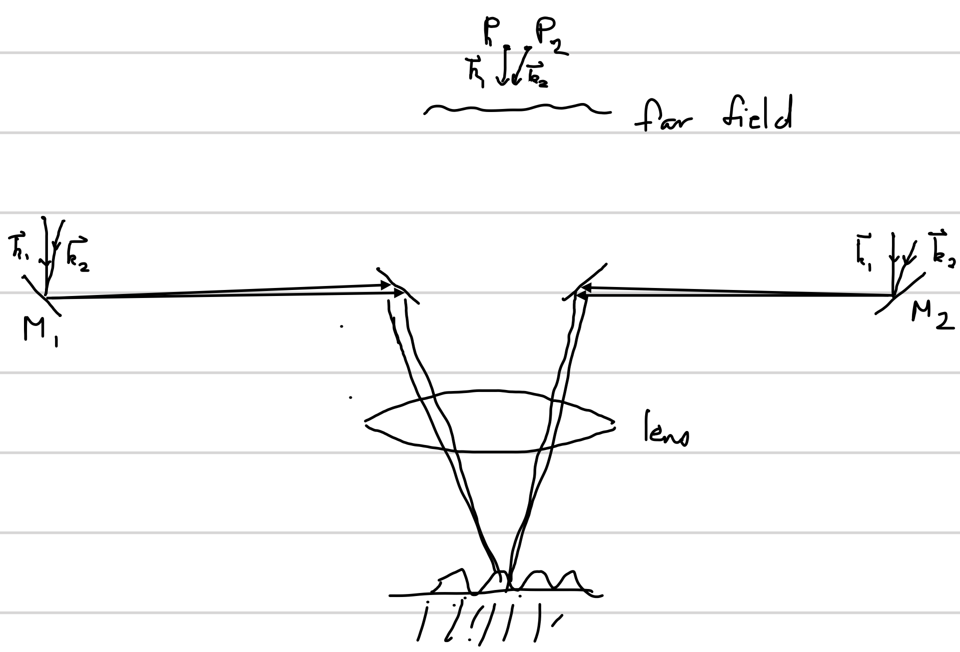
The visibility of the fringes is defined

$$V = \frac{I_{max}(\tau) - I_{max}(\tau)}{I_{max}(\tau) + I_{max}(\tau)} = \gamma(\tau)$$

Thus, by measuring the visibility of the fringes, one can determine the coherence time and spectral density of the signal.

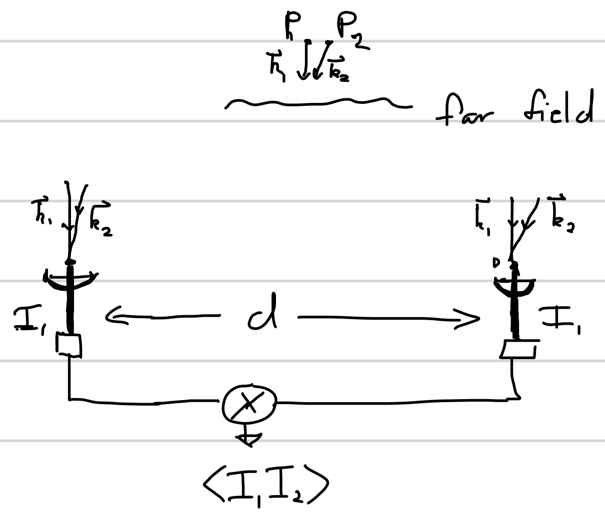
Hanbury-Brown & Twiss Effect

An important development in our understanding of quantum coherence of electromagnetic fields was initiated by Hanbury-Brown & Twiss (1956) who considered a new method for measuring coherence based on intensity-correlation, rather than field correlation. Their interest was in the coherence between two points in space, rather than the coherence at two instants of time at the same position.



Sketched above is a two-path interferometer known as a Michaelson stellar interferometer. If the points P_1 and P_2 are "coherent," i.e, the phase of oscillation of P_1 is correlated with P_2, then we see high visibility fringes. If, on the other hand, P_1 and P_2 are not correlated, then when M_1 + M_2 are sufficiently separated, the visibility will decrease – this separation determines the "coherence length" of the source (analogous to the temporal coherence). It can be used to determine the angular size of a star or to distinguish a "double star" from a single source. This type of interferometer, like the Mach-Zender, measures the interference between field amplitudes. But it is not stable. It is very sensitive to vibrations on the mirrors, and any phase fluctuations in the paths, as might occur due fluctuations in the path length as light passes through the atmosphere.

Hanbury-Brown & Twiss considered measuring spatial coherence by directly measuring the intensity, at two positions and <u>then</u> correlating the result, rather than bringing the two fields together and then measuring the intensity (and thus the interference between paths).

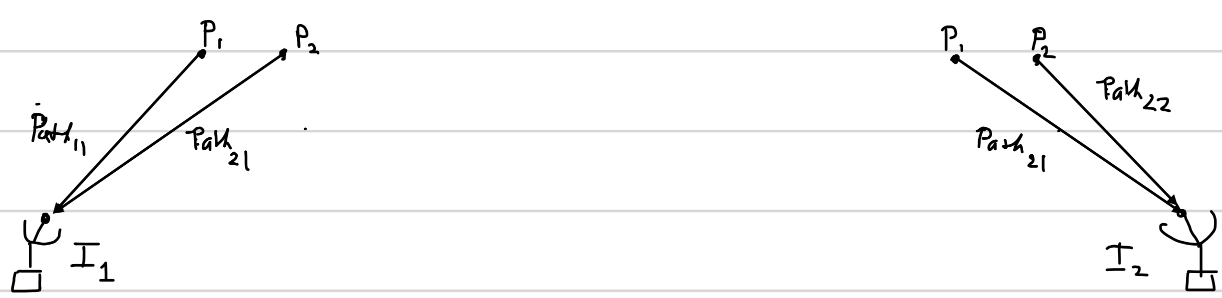


HBT claimed that $\langle I_1 I_2 \rangle$ would exhibit the correlation between P_1 & P_2 as a function of the separation d of the two antennas. The argument is straightforward. The intensity $I = \langle I \rangle + \Delta I$ where $\langle I \rangle$ is the expected value and ΔI is the deviation from the mean (a "fluctuation"). Thus,

$$\langle I_1 I_2 \rangle = \langle (\langle I_1 \rangle + \Delta I_1)(\langle I_2 \rangle + \Delta I_2) \rangle = \langle I_1 \rangle \langle I_2 \rangle + \langle \Delta I_1 \Delta I_2 \rangle$$

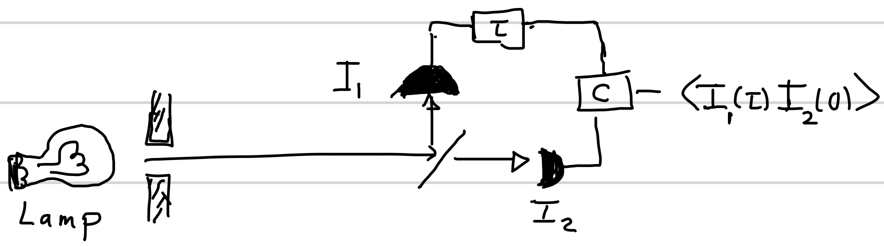
If the two intensities are uncorrelated, then $\langle I_1 I_2 \rangle = \langle I_1 \rangle \langle I_2 \rangle$, so the deviation,
$\langle I_1 I_2 \rangle - \langle I_1 \rangle \langle I_2 \rangle = \langle \Delta I_1 \Delta I_2 \rangle$ is a measure of the <u>correlation</u> between the two signals, which should fall off with the separation d, and thus allow one to measure the coherence length.

This caused quite a controversy, because any one photon takes only one path to arrive at detector 1 or 2, and thus an individual photon does not "interfere with itself."

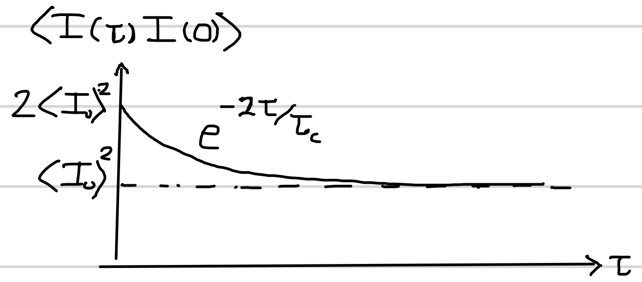


There are not two paths of any one photon that lead to detection in both I_1 and I_2

To test these ideas, HBT considered the temporal coherence equivalent experiment



Two detectors are placed symmetrically on either side of a 50-50 beam-splitter. The signal from detector-1 is correlated with detector-2 after a delay τ. The result is the correlation between the intensity of the field entering the beam-splitter at two different times. Again, any photon will either be transmitted or reflected, but the correlation is not due to the interference of two paths that any individual photons. Nonetheless, HBT measured the following signal



$$\langle I_0^2\rangle = \langle I(0) I(0)\rangle$$
$$= \langle I(\tau) I(\tau)\rangle \quad \text{(stationary statistics)}$$

For short time, the intensity correlation is twice as large as the asymptotic value, with the decay given by ½ the correlation time seen in the visibility of the Mach-Zehnder interferometer.

The HBT effect can be seen in the classical statistical correlations of random fluctuating waves. Consider the correlation function:

$$\langle I(\tau) I(0)\rangle = \langle E^*(\tau) E(\tau) E^*(0) E(0)\rangle = \int d[E]\, P[E]\, E^*(\tau) E(\tau) E^*(0) E(0)$$

We can simplify this expression given the known probability distribution for the field of "natural light." We found a Gaussian probability distribution for the complex amplitude of a given mode

$$P(\alpha_k) = \frac{1}{\pi N} e^{-\frac{|\alpha_k|^2}{N}} \Rightarrow P(\{\alpha_k\}) = \prod_k P(\alpha_k) = \frac{1}{\pi N} e^{-\frac{\sum_k |\alpha_k|^2}{N}}$$

Thus $P[E]$ is also Gaussian. This allows us to calculate all moments of the distribution, i.e., all expectation values of polynomials in E

Wick's Theorem: Moments of a Gaussian probability distribution.

$$\langle E^*(x_1)E^*(x_2)\cdots E^*(x_n)E(x_1)E(x_2)\cdots E(x_n)\rangle = \underbrace{\sum_{\text{pairs}}\prod_{ij}\langle E^*(x_i)E(x_j)\rangle}_{\text{Sum of all possible pairings of } E^*(x_i) \text{ with } E(x_j)}$$

Thus, $\langle I(\tau)I(0)\rangle = \langle E^*(\tau)E^*(0)E(\tau)E(0)\rangle = \langle E^*(\tau)E(\tau)\rangle\langle E^*(0)E(0)\rangle + \langle E^*(\tau)E(0)\rangle\langle E^*(0)E(\tau)\rangle$

$$= \langle I(\tau)\rangle\langle I(0)\rangle + |\gamma(\tau)|^2 = \langle I_0\rangle^2 + |\langle I_0\rangle e^{-\tau/\tau_c}|^2 = \langle I_0^2\rangle + \langle I_0^2\rangle e^{-2\tau/\tau_c}$$

$$\Rightarrow \boxed{\langle I(\tau)I(0)\rangle = \langle I_0^2\rangle\left(1 + \frac{|\langle E^*(0)E(\tau)\rangle|^2}{\langle I_0\rangle^2}\right) = \langle I_0\rangle^2\left(1 + e^{-2\tau/\tau_c}\right)}$$

The intensity-intensity correlation function allows us to measure to temporal coherence auto-correlation function: $\langle E^*(0)E(\tau)\rangle$.

Note: At $\tau=0$: $\langle I(0)I(0)\rangle - \langle I(0)\rangle\langle I(0)\rangle = \langle I_0^2\rangle - \langle I_0\rangle^2 = \langle \Delta I^2\rangle$. Thus at $\tau=0$, the enhancement of the intensity-intensity correlation over the uncorrelated product is the intensity fluctuation.

For natural light, $P(I)\ \frac{1}{\langle I_0\rangle}e^{-I/\langle I_0\rangle} \Rightarrow \langle I^n\rangle = n!\langle I_0\rangle^n \Rightarrow \langle I^2\rangle = 2\langle I_0\rangle^2$

(2 ↑ twice uncorrelated product.)

$$\Rightarrow \langle \Delta I^2\rangle = \langle I^2\rangle - \langle I\rangle^2 = \langle I_0\rangle^2$$

Note: $\langle I(\tau)I(0)\rangle - \langle I(\tau)\rangle\langle I(0)\rangle = \langle \Delta I(\tau)\Delta I(0)\rangle$: Correlation of fluctuations.

When $\tau \ll \tau_c$, we see fluctuations of I that are correlated. For $\tau \gg \tau_c$, the fluctuations are not correlated, and we see the product of the average intensity.

Quantum Theory of photon counting: Glauber Correlation Functions

The classical stochastic wave theory explains the HBT effect. But the question of the quantum explanation remains. The electromagnetic field is has wave-particle duality. How does one explain the HBT effect from the point of view of photon paths?

To answer the question we turn to the fully quantum theory, as developed by Roy Glauber, that led to the modern theory of quantum optics, for which he was awarded the Nobel Prize in 2005. Let us return to the photoelectric effect, but now in the fully quantum theory, including the quantized electromagnetic field.

Consider a one atom detector. The interaction Hamiltonian is taken as the dipole interaction

$$\hat{H} = -\hat{\vec{d}}\cdot\hat{\vec{E}}(\vec{r},t) = -\hat{\vec{d}}\cdot\hat{\vec{E}}^{(+)}(\vec{r},t) - \hat{\vec{d}}\cdot\hat{\vec{E}}^{(-)}(\vec{r},t)$$

We seek the transition probability $|g\rangle \Rightarrow |e\rangle$, where $|e\rangle$ is in the continuum for the atom. The electron of the photoionized atom is ultimately measured; the state of the field after photo-ionization is not measured.

Let $|\Psi^{AF}_{initial}\rangle = |g\rangle \otimes |\psi^{in}_F\rangle$, $|\Psi^{AF}_{final}\rangle = |e\rangle \otimes |\psi^{out}_F\rangle$. By Fermi's Golden Rule, the transition probability to photo-ionize the atom is proportional to

$$p^{(1)} \propto \sum_{\psi^{out}_F} |\langle e|d|g\rangle|^2 |\langle \psi^{out}_{field}|\hat{E}^{(+)}(\vec{r},t)|\psi^{in}_{field}\rangle|^2$$

$$\Rightarrow p^{(1)} \propto \sum_{\psi^{out}_F} \langle \psi^{in}_{field}|\hat{E}^{(-)}(\vec{r},t)|\psi^{out}_{field}\rangle\langle \psi^{out}_{field}|\hat{E}^{(+)}(\vec{r},t)|\psi^{in}_{field}\rangle$$

(I ignore the polarization of the field under the assumption we drive a given dipole transition)

The sum over final field state can then be extended to a sum over all states over the field since additional states not connected by the photo-ionization process will have zero matrix element $\langle \psi^{out}_{field}|\hat{E}^{(+)}(\vec{r},t)|\psi^{in}_{field}\rangle$

$$\Rightarrow p^{(1)} \propto \langle \hat{E}^{(-)}(\vec{r},t)\,\hat{E}^{(+)}(\vec{r},t)\rangle$$

This is the quantum version of the semiclassical theory of photon counting:

Probability to count a photon in short time Δt:

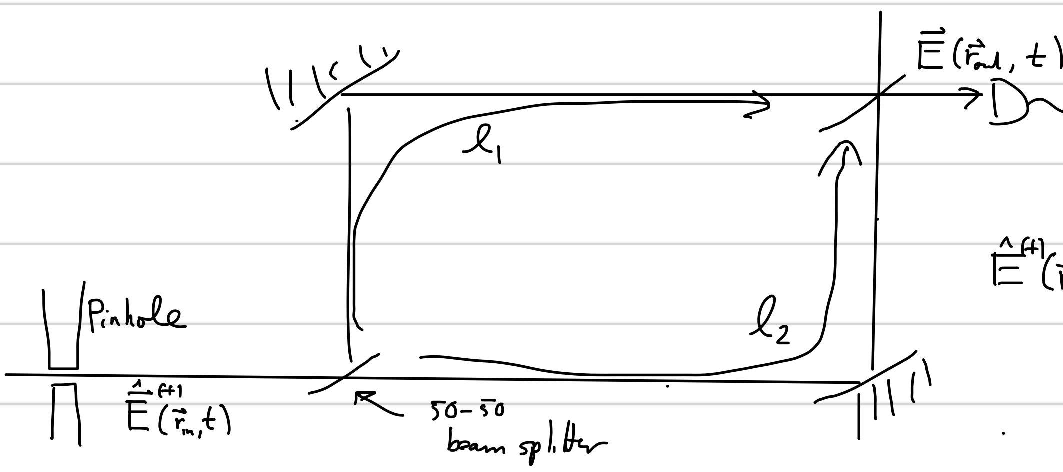
Semiclassical: $p^{(1)} = \eta\langle I(\vec{r},t)\rangle\Delta t = \eta \int d[\mathcal{E}]\, |\mathcal{E}(\vec{r},t)|^2\, \Delta t$

Quantum: $p^{(1)} = \eta \langle \hat{I}(\vec{r},t)\rangle_\rho \Delta t$: $\hat{I}(\vec{r},t) = \hat{E}^{(-)}(\vec{r},t)\,\hat{E}^{(+)}(\vec{r},t)$ (intensity operator)

expected value over incident field

Quantum theory of "first-order" interference

Consider again the Mach-Zender interferometer. The configuration is exactly the same as we saw before:



$$\langle \hat{I}_{out} \rangle = \langle \hat{E}^{(-)}(\vec{r}_{out}, t)\, \hat{E}^{(+)}(\vec{r}_{out}, t) \rangle$$

$$\hat{E}^{(+)}(\vec{r}_{out}, t) = \frac{1}{2}\left(\hat{E}^{(+)}(\vec{r}_{in}, t_1) + \hat{E}^{(+)}(\vec{r}_{in}, t_2) \right)$$

$$t_1 = t - \frac{l_1}{c}, \qquad t_2 = t - \frac{l_2}{c}$$

$$\Rightarrow \langle \hat{I}_{out} \rangle = \frac{1}{4}\left[\langle \hat{E}^{(-)}(t_1) \hat{E}^{(+)}(t_1) \rangle + \langle \hat{E}^{(-)}(t_2)\, \hat{E}^{(+)}(t_2) \rangle + \langle \hat{E}^{(-)}(t_1)\, \hat{E}^{(+)}(t_2) \rangle + \langle \hat{E}^{(-)}(t_2)\, \hat{E}^{(+)}(t_1) \rangle \right]$$ (Suppressing position dependence, since all at same $\vec{r}$)

$$= \frac{1}{2}\left[\langle \hat{I}_{in} \rangle + \mathrm{Re} \langle \hat{E}^{(-)}(0)\, \hat{E}^{(+)}(\tau) \rangle \right]$$

where under the assumption of stationary statistics: $\langle \hat{I}_{in} \rangle = \langle \hat{E}^{(-)}(t_1)\, \hat{E}^{(+)}(t_1) \rangle = \langle \hat{E}^{(-)}(t_2)\, \hat{E}^{(+)}(t_2) \rangle$

$$\langle \hat{E}^{(-)}(t_1)\, \hat{E}^{(+)}(t_2) \rangle = \langle \hat{E}^{(-)}(0)\, \hat{E}^{(+)}(\tau) \rangle, \quad \tau = t_2 - t_1$$

The expected value of the intensity measured at the output port of the interferometer takes exactly the same form as the classical expression. We recover exactly the classical expression when the state of a field in a coherent state.

$$\hat{E}^{(+)}(\vec{r}, t) \,|\{\alpha_k\}\rangle = \mathcal{E}(\vec{r}, t) \,|\{\alpha_k\}\rangle$$

Quasiclassical field $\mathcal{E}(\vec{r}, t) = \sum_k \sqrt{\frac{2\pi\hbar\omega_k}{V}}\, \alpha_k\, e^{i(\vec{k}\cdot\vec{r} - \omega_k t)}$

A coherent state with stationary statistics is monochromatic $\mathcal{E} = \mathcal{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$ (single mode)

$$\Rightarrow \langle \hat{I}_{out} \rangle = \frac{1}{2}\left[|\mathcal{E}_0|^2 + |\mathcal{E}_0|^2 \cos\omega_0\tau \right] = \frac{\langle \hat{I}_0 \rangle}{2}\left(1 + \cos\omega_0\tau \right)$$

$|\mathcal{E}_0|^2 \Rightarrow \langle \hat{I}_0 \rangle$; Perfectly coherent field

Within the quantum theory, we can interpret this a interference, photon by photon. For a single mode

$$\hat{E}^{(+)}(\vec{r}_{out}, t) = \sqrt{\frac{2\pi\hbar\omega}{V}} \left(\frac{\hat{a}_k e^{-i\omega_k t_1} + \hat{a}_k e^{-i\omega_k t_2}}{2} \right) e^{i\vec{k}\cdot\vec{r}_{in}} \Rightarrow \hat{I}(\vec{r}, t) \propto \hat{a}_k^{\dagger}\hat{a}_k \quad \text{\# of photons in given mode}$$

$$\Rightarrow \hat{I}_{out} \propto \frac{1}{2}\langle \hat{a}_k^{\dagger}\hat{a}_k \rangle + \frac{1}{4}\left\langle (\hat{a}_k e^{i\omega t_1})^{\dagger} (\hat{a}_k e^{-i\omega t_2}) \right\rangle + \frac{1}{4}\left\langle (\hat{a}_k e^{-i\omega t_2})^{\dagger} (\hat{a}_k e^{-i\omega t_1}) \right\rangle$$

This expression shows the interference arises single photon by single photon. The same interference pattern would be seen by repeatedly sending single photons into the interferometer:

$$|\psi^{in}_{field}\rangle = |1_{\vec{k}}\rangle \Rightarrow \langle (\hat{a}e^{-i\omega t_1})^\dagger (\hat{a}e^{-i\omega t_2})\rangle = e^{-i\omega\tau}$$

⇒ Mach-Zender two path-interferometer: Coherence seen in the interference of probability amplitudes of the paths of individual photons.

General recovery of classical statistical fluctuations of partially coherent light:

Mixed state: $\hat{\rho} = \int d^2\{\alpha_k\}\, P(\{\alpha_k\})\, |\{\alpha_k\}\rangle\langle\{\alpha_k\}|$ (Statistical mixture of coherent states)

↑ real, positive, probability distribution

$$\langle \hat{E}^{(-)}(t_1)\,\hat{E}^{(+)}(t_2)\rangle = Tr(\hat{E}^{(-)}(t_2)\,\hat{E}^{(+)}(t_1)\,\hat{\rho}) = \underbrace{\int d^2\{\alpha_k\}\, P(\{\alpha_k\})\, \mathcal{E}^*(t_1)\,\mathcal{E}(t_2)}_{\text{Classical statistical average}}$$

$$\Rightarrow \langle \hat{I}_{out}\rangle = \frac{1}{2}\left[\overline{|\mathcal{E}|^2} + Re(\overline{\mathcal{E}^*(t_1)\mathcal{E}(t_2)})\right] = \frac{1}{2} I_0 \left(1 + |\gamma(\tau)| \cos\omega_0\tau\right)$$

$\gamma(\tau)$ ↑ coherence function = Fourier transform of spectral density

For natural light: $P(\{\alpha_k\}) = \prod_k \frac{1}{\pi\langle|\alpha_k|^2\rangle} e^{-\frac{|\alpha_k|^2}{\langle|\alpha_k|^2\rangle}} = \prod_k \frac{1}{\pi\langle\hat{n}_k\rangle} e^{-\frac{|\alpha_k|^2}{\langle\hat{n}_k\rangle}}$

Where $\langle\hat{n}_k\rangle = \overline{|\alpha_k|^2}$ = average # of photons in the mode

$$\Rightarrow |\gamma(\tau)| = e^{-\tau/\tau_c}, \quad \text{as before}$$

Bose-Einstein distribution:

We have seen that "natural light" is represented by Gaussian fluctuations of the wave amplitude. We can also consider the representation of this mixed state in the number basis.

I will leave it as an exercise to show, for a given mode,

$$\int d^2\alpha_k \frac{1}{\pi\langle\hat{n}_k\rangle} e^{-\frac{|\alpha_k|^2}{\langle\hat{n}_k\rangle}} |\alpha_k\rangle\langle\alpha_k| = \sum_{n_k=0}^{\infty} \frac{\langle\hat{n}_k\rangle^{n_k}}{(1+\langle\hat{n}_k\rangle)^{n_k+1}} |n_k\rangle\langle n_k|$$

↑ "continuous variable" (waves) — ↑ discrete variable (particles)

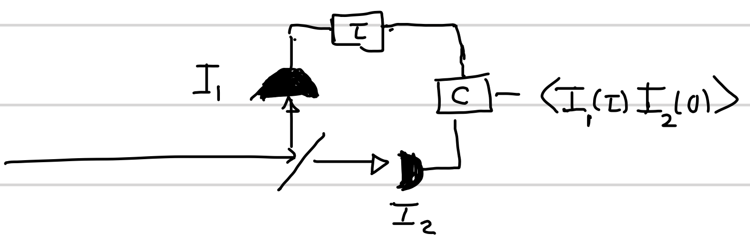
The probability distribution of photon excitations: $P(n_k) = \frac{\langle \hat{n}_k \rangle^{n_k}}{(1+\langle \hat{n}_k \rangle)^{n_k+1}}$ is the famous Bose-Einstein distribution associated with a state of identical bosons (here photons) with average number $\langle \hat{n}_k \rangle$. There is an "effective temperature" $\langle \hat{n}_k \rangle = \frac{1}{e^{\frac{\hbar\omega_k}{k_B T}} - 1}$

We often use the term "thermal state" to represent the natural state of light, such as collision broadened light from a gas of atoms. Thermal light exhibits "first order coherence" with a coherence time depending on the power spectrum

$$\langle \hat{I}_{out} \rangle = \frac{\langle \hat{n}_k \rangle}{2} \left(1 + |\gamma(\tau)| \cos \omega_0 \tau \right)$$

Higher order correlations: Coincidence Counting

The Hanbury-Brown & Twiss effect differs fundamentally from the Mach-Zender-type first order interference effect in that it involves correcting intensities rather than field amplitudes, and thus involves the joint-probability for detecting more than one photon. For example, the temporal HBT effect



We can think about this as a photon counting experiment. The correlator, C, goes "click" if a photo-electron is ejected in detector-1 and one in detector-2, separated by time τ. This is a two-photon correlation. Glauber showed using the generalization of Fermi's Golden Rule, that the joint prob. of detection is proportional to:

$$P^{(2)} \propto \sum_{\psi_{final}} \left| \langle \psi_{out}^{field} | \hat{E}^{(+)}(\vec{r}, \tau) \, \hat{E}^{(+)}(\vec{r}, 0) \, | \psi_{in}^{field} \rangle \right|^2$$

annihilate photon at time τ, annihilate photon at time 0

$$= \langle \psi_{in} | \hat{E}^{(-)}(\vec{r}, \tau) \, \hat{E}^{(-)}(\vec{r}, 0) \, \hat{E}^{(+)}(\vec{r}, \tau) \, \hat{E}^{(+)}(\vec{r}, 0) \, | \psi_{in} \rangle$$

We can write this compactly as: $P^{(2)} \propto \langle : \hat{I}(\tau) \hat{I}(0) : \rangle$, where the double dots stand for "normal order." Normal order means, in the order where all creation operators are on right and all annihilation operators are on left. (Nothing is commuted)

Intensity - Intensity Correlation ⇒ Two photon correlations

Semiclassical HBT: $\langle I(\tau) I(0)\rangle = \int d\{\alpha_k\}\, P(\{\alpha_k\})\, \mathcal{E}^*(\tau)\,\mathcal{E}(\tau)\,\mathcal{E}^*(0)\,\mathcal{E}(0)$

Quantum HBT: $\langle : \hat{I}(\tau)\,\hat{I}(0) : \rangle = \langle \hat{E}^{(-)}(\tau)\,\hat{E}^{(-)}(0)\,\hat{E}^{(+)}(\tau)\,\hat{E}^{(+)}(0)\rangle = \mathrm{Tr}\left(\hat{E}^{(-)}(\tau)\,\hat{E}^{(-)}(0)\,\hat{E}^{(+)}(\tau)\,\hat{E}^{(+)}(0)\,\hat{\rho}\right)$

For classical statistical fluctuations $\hat{\rho} = \int d\{\alpha_k\}\, P(\{\alpha_k\})\, |\{\alpha_k\}\rangle\langle\{\alpha_k\}|$

$\langle : \hat{I}(\tau)\,\hat{I}(0) : \rangle = \int d\{\alpha_k\}\, P(\{\alpha_k\})\, |\mathcal{E}(\tau)|^2\, |\mathcal{E}(0)|^2$ ← Exactly the semiclassical result

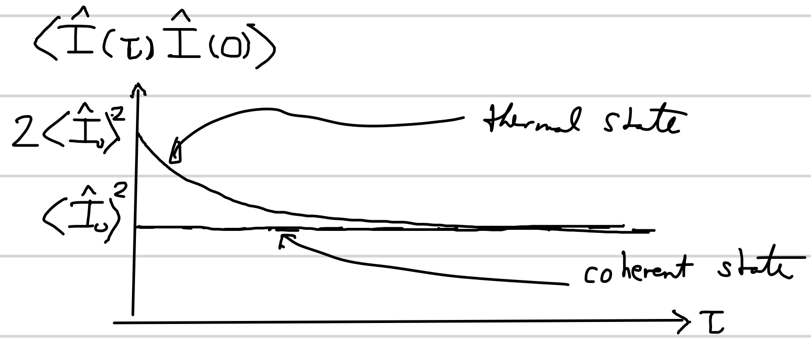
For a "thermal state" the fully quantum theory is exactly the same as the classical prediction

For Gaussian fluctuations in the wave amplitude:

$$\langle \hat{E}^{(-)}(\tau)\,\hat{E}^{(-)}(0)\,\hat{E}^{(+)}(\tau)\,\hat{E}^{(+)}(0)\rangle = \underbrace{\langle \hat{E}^{(-)}(\tau)\,\hat{E}^{(+)}(\tau)\rangle}_{\langle \hat{I}_0\rangle}\underbrace{\langle \hat{E}^{(-)}(0)\,\hat{E}^{(+)}(0)\rangle}_{\langle \hat{I}_0\rangle} + \underbrace{\langle \hat{E}^{(-)}(\tau)\,\hat{E}^{(+)}(0)\rangle\langle \hat{E}^{(-)}(0)\,\hat{E}^{(+)}(\tau)\rangle}_{\langle \hat{I}_0\rangle^2 |\gamma(\tau)|^2}$$

Note: For a coherent state $P(\{\alpha_k\}) = \delta(\{\alpha_k - \alpha_k^0\})$

$\langle \hat{E}^{(-)}(\tau)\,\hat{E}^{(-)}(0)\,\hat{E}^{(+)}(\tau)\,\hat{E}^{(+)}(0)\rangle = \mathcal{E}^*(\tau)\,\mathcal{E}^*(0)\,\mathcal{E}(\tau)\,\mathcal{E}(0) = |\mathcal{E}(\tau)|^2\,|\mathcal{E}(0)|^2 = I(\tau) I(0) = \langle \hat{I}_0\rangle^2$: Independent of τ



General Correlation Functions

The joint probability to detect n-photons at n-space/time points $x = (\vec{r}, t)$

$$p^{(n)}(x_1, x_2, \ldots, x_n) \propto \langle : \hat{I}(x_1)\,\hat{I}(x_2)\cdots\hat{I}(x_n) : \rangle = \langle \hat{E}^{(-)}(x_1)\,\hat{E}^{(-)}(x_2)\cdots\hat{E}^{(-)}(x_N)\,\hat{E}^{(+)}(x_1)\,\hat{E}^{(+)}(x_2)\cdots\hat{E}^{(+)}(x_N)\rangle$$

Define: $G^{(n)}(x_1, x_2, \ldots, x_n, x_1', x_2', \ldots, x_n') = \langle \hat{E}^{(-)}(x_1)\,\hat{E}^{(-)}(x_2)\cdots\hat{E}^{(-)}(x_n)\,\hat{E}^{(+)}(x_1')\,\hat{E}^{(+)}(x_2')\cdots\hat{E}^{(+)}(x_n')\rangle$

A field, is said to be n^{th}-order coherent if the n^{th}-order correlation function factories

- First-order coherence: Depends only only spectral density: Coherent = narrow band (e.g. laser, single photon states in single mode, filtered thermal light)
- Second order coherence: depends on quantum statistics
- A coherent state $|\{\alpha_k\}\rangle$ exhibits "coherence" to all orders.

Normalized Correlation function:

$$g^{(n)}(x_1, x_2, \cdots, x_n; x_1', x_2', \cdots x_n') = \frac{G^{(n)}(x_1, x_2, \cdots, x_n; x_1', x_2', \cdots, x_n')}{\left[G^{(1)}(x_1,x_1)\, G^{(1)}(x_2, x_2) \cdots G^{(1)}(x_n, x_n)\; G^{(1)}(x_1', x_1')\; G^{(1)}(x_2', x_2') \cdots G^{(1)}(x_n', x_n')\right]^{1/2}}$$

The n-fold coincidence function:

$$g^{(n)}(x_1, x_2, \cdots x_n) = \frac{G^{(n)}(x_1, x_2 \cdots x_n;\; x_1, x_2, x_n)}{G^{(1)}(x_1,x_1)\, G^{(1)}(x_2, x_2) \cdots G^{(1)}(x_n, x_n)}$$

For a coherent state: $g^{(n)}(x_1, x_2, \cdots; x_n) = 1$

First-order coherence:
$$\langle \hat{I}_{out} \rangle = \frac{1}{4}\left[\langle \hat{E}^{(-)}(t_1)\, \hat{E}^{(+)}(t_1)\rangle + \langle \hat{E}^{(-)}(t_2)\, \hat{E}^{(+)}(t_2)\rangle + \langle \hat{E}^{(-)}(t_1)\, \hat{E}^{(+)}(t_2)\rangle + \langle \hat{E}^{(-)}(t_2)\, \hat{E}^{(+)}(t_1)\rangle\right]$$
$$= \frac{1}{4}\left[G^{(1)}(t_1,t_1) + G^{(1)}(t_2, t_2) + G^{(1)}(t_1, t_2) + G^{(2)}(t_2, t_2)\right]$$
$$= \frac{1}{2}\langle \hat{I}_0\rangle \left[1 + \mathrm{Re}\,[\underbrace{g^{(1)}(t_1, t_2)}_{\equiv g^{(1)}(\tau)}]\right] \quad \text{(for stationary statistics)}$$
$$\tau = t_2 - t_1$$

Second-Order coherence:
$$\langle : \hat{I}(\tau)\hat{I}(0) : \rangle = \langle \hat{E}^{(-)}(\tau)\, \hat{E}^{(-)}(0)\, \hat{E}^{(+)}(\tau)\, \hat{E}^{(+)}(0) \rangle \equiv G^{(2)}(\tau)$$

$$g^{(2)}(\tau) \equiv \frac{\langle \hat{E}^{(-)}(\tau)\, \hat{E}^{(-)}(0)\, \hat{E}^{(+)}(\tau)\, \hat{E}^{(+)}(0)\rangle}{\langle \hat{E}^{(-)}(\tau)\, \hat{E}^{(+)}(\tau)\rangle\langle \hat{E}^{(-)}(0)\, \hat{E}^{(+)}(0)\rangle} = \frac{G^{(2)}(\tau)}{\langle \hat{I}_0\rangle^2}$$

Thermal light
$$G^{(2)}(\tau) = \underbrace{\langle \hat{E}^{(-)}(\tau)\, \hat{E}^{(+)}(\tau)\rangle\langle \hat{E}^{(-)}(0)\, \hat{E}^{(+)}(0)\rangle}_{\langle \hat{I}_0\rangle^2} + \langle \hat{E}^{(-)}(0)\, \hat{E}^{(+)}(\tau)\rangle\langle \hat{E}^{(-)}(\tau)\, \hat{E}^{(+)}(0)\rangle = \langle \hat{I}_0\rangle^2\, g^{(2)}(\tau)$$

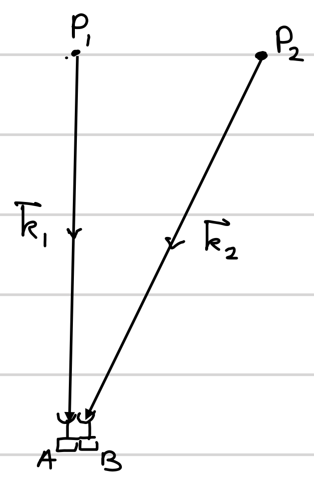
$$g^{(2)}(\tau) = 1 + |g^{(1)}(\tau)|^2$$

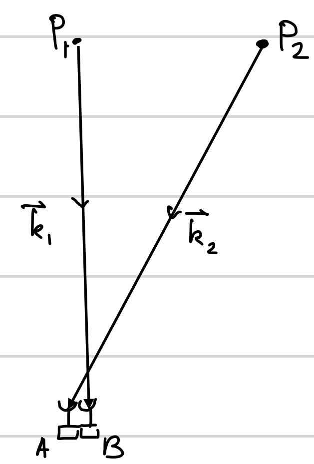
Coherent state $\quad G^{(2)}(\tau) = \langle \hat{I}_0\rangle^2 \Rightarrow g^{(2)}(\tau) = 1$

HBT – The particle interference picture

We now come to the heart of the matter. We have explained the HBT effect in terms correlations of fluctuations of stochastic wave amplitudes. But how do we describe the effect in terms of photons?

Consider again the spatial coherence problem originally studied by HBT. We seek to distinguish a double star from a single star. The light that hits the antennas that are very close is an incoherent mixture of two possible modes. Each antenna detects a photon that took one possible path, but we do not which of the two possible paths the two photons took





When the two detectors at at the same position, these two processes are indistinguishable. In principle there is no way to know whether photon-1 arrived at detector-A and photon-2 at detector-B or vice versa. Thus these two histories interfere. This kind of two-photon interference is seen in the Glauber correlation functions:

$$G^{(2)}(\vec{r}_A, \vec{r}_B) = \langle \hat{E}^{(-)}(\vec{r}_A)\, \hat{E}^{(-)}(\vec{r}_B)\, \hat{E}^{(+)}(\vec{r}_A)\, \hat{E}^{(+)}(\vec{r}_B) \rangle$$

Let us take the state to be two photons, each described by some wavepacket with momentum-space wave functions $\tilde{\psi}_k$ and $\tilde{\varphi}_k$ respectively: $|\Psi\rangle = \hat{a}^\dagger[\psi]\, \hat{a}^\dagger[\phi]\, |0\rangle$, $\hat{a}^\dagger[\psi] = \sum_k \tilde{\psi}_k \hat{a}^\dagger_k$, $\hat{a}^\dagger[\phi] = \sum_k \tilde{\varphi}_k \hat{a}^\dagger_k$. Note: $[\hat{a}_{\vec{k}}, \hat{a}^\dagger[\psi]] = \tilde{\psi}_k$, $[\hat{a}_k, \hat{a}^\dagger[\varphi]] = \tilde{\varphi}_k$.

$$\Rightarrow G^{(2)}(\vec{r}_1, \vec{r}_2) = \langle \Psi | \hat{E}^{(-)}_{(\vec{r}_A)}\, \hat{E}^{(-)}_{(\vec{r}_B)}\, \hat{E}^{(+)}_{(\vec{r}_A)}\, \hat{E}^{(+)}_{(\vec{r}_B)} | \Psi \rangle = \sum_{\{n_k\}} \langle \Psi | \hat{E}^{(-)}_{(\vec{r}_A)}\, \hat{E}^{(-)}_{(\vec{r}_B)} |\{n_k\}\rangle\langle\{n_k\}| \hat{E}^{(+)}_{(\vec{r}_A)}\, \hat{E}^{(+)}_{(\vec{r}_B)} | \Psi \rangle$$

where I have inserted a complete set of Fock states. Now $\hat{E}^{(+)}_{(\vec{r})} = \sum_{\vec{k}} \sqrt{\frac{2\pi\hbar\omega_k}{V}}\, e^{i\vec{k}\cdot\vec{r}}\, \hat{a}_k$ will annihilate one photon. So $\hat{E}^{(+)}_{(\vec{r}_A)} \hat{E}^{(+)}_{(\vec{r}_A)} |\Psi\rangle \propto |0\rangle$, the vacuum.

Thus
$$G^{(2)}(\vec{r}_A, \vec{r}_B) = \underbrace{|\langle 0 | \hat{E}^{(+)}(\vec{r}_A)\, \hat{E}^{(+)}(\vec{r}_B) | \Psi \rangle}|^2 = |\Psi(\vec{r}_A, \vec{r}_B)|^2$$

$\equiv \Psi(\vec{r}_A, \vec{r}_B)$, the effective "two-photon wave function"

$$\Psi(\vec{r}_A, \vec{r}_B) = \sum_{k,k'} \frac{2\pi\hbar}{V}\sqrt{\omega_k \omega_{k'}}\, e^{i\vec{k}\cdot\vec{r}_A} e^{i\vec{k}'\cdot\vec{r}_B} \underbrace{\langle 0|\hat{a}_{\vec{k}}\hat{a}_{\vec{k}'}\hat{a}^\dagger[\psi]\hat{a}^\dagger[\varphi]|0\rangle}$$

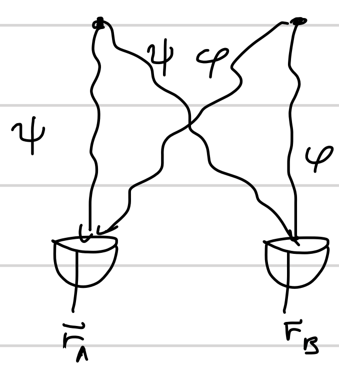
$$= \langle 0|[\hat{a}_{\vec{k}}\hat{a}_{\vec{k}'}, \hat{a}^\dagger[\psi]\hat{a}^\dagger[\varphi]]|0\rangle$$

Aside $\langle 0|[\hat{a}_{\vec{k}}\hat{a}_{\vec{k}'}, \hat{a}^\dagger[\psi]\hat{a}^\dagger[\varphi]]|0\rangle = \langle 0|\hat{a}_{\vec{k}}[\hat{a}_{\vec{k}'}, \hat{a}^\dagger[\psi]]\hat{a}^\dagger[\varphi]|0\rangle + \langle 0|\hat{a}_{\vec{k}'}\hat{a}^\dagger[\psi][\hat{a}_{\vec{k}}, \hat{a}^\dagger[\varphi]]|0\rangle$

$$= \tilde{\psi}_{\vec{k}'}\langle 0|\hat{a}_{\vec{k}}\hat{a}^\dagger[\varphi]|0\rangle + \tilde{\varphi}_{\vec{k}'}\langle 0|\hat{a}_k\hat{a}^\dagger[\psi]|0\rangle = \tilde{\psi}_{\vec{k}'}\tilde{\varphi}_{\vec{k}} + \tilde{\varphi}_{\vec{k}'}\tilde{\psi}_{\vec{k}}$$

$$\Rightarrow \Psi(\vec{r}_A, \vec{r}_A) = \tilde{E}_\psi(\vec{r}_A)\tilde{E}_\varphi(\vec{r}_B) + \tilde{E}_\varphi(\vec{r}_A)\tilde{E}_\psi(\vec{r}_B)$$

where $\tilde{E}_\psi(\vec{r}) \equiv \sum_{\vec{k}} \sqrt{\frac{2\pi\hbar\omega_k}{V}}\, \tilde{\psi}_{\vec{k}}\, e^{i\vec{k}\cdot\vec{r}}$ is the effective electric field associated with the photon wave packet. It plays the role of the "photon wave function." What we see here is that because the photons are bosons the two photon wave function is symmetrized. Thus there are two histories associated with joint detection at positions $\vec{r}_A$ and $\vec{r}_B$: Photon in wavepacket ψ is detected at $\vec{r}_A$ and photon in wavepacket φ at $\vec{r}_B$ and viceversa.

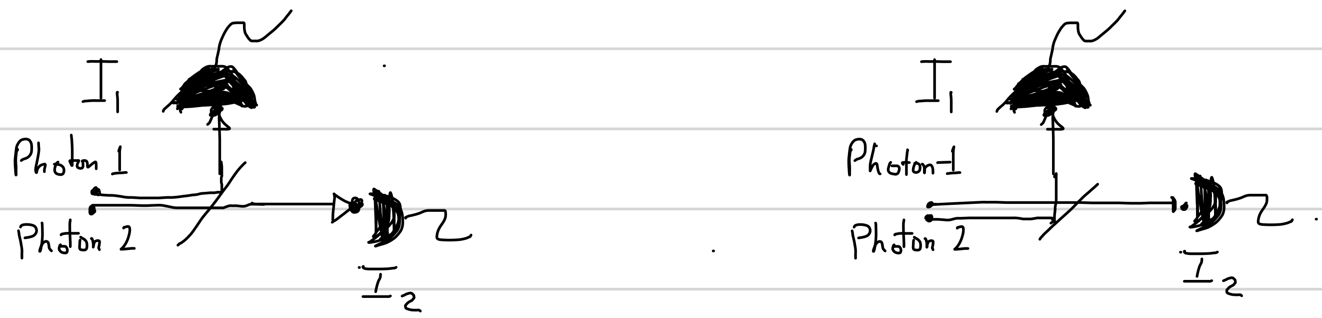


We must add (symmetrically), the two probability amplitudes for these two histories to get the total probability amplitude for joint detection, and then square to get the probability.

The Hanbury-Brown and Twiss experiment was the first example of two-photon interference. It does not contradict Dirac's statement. This was only in the context of first-order interference. First order interference is essentially a one-photon effect: every photon only interferences with itself. However, in second order, involving coincidence counting, two-photon histories interfere.

Photon statistics and temporal coherence

The HBT effect seen in the temporal coherence of a single spatial mode can be understood in terms of the quantum state of the field and its representation in terms of photon number.



Again, there are two possible histories that lead to a coincidence count. For zero time delay, the rate of coincidence count is proportional to

$$\langle : \hat{I}(0)\hat{I}(0): \rangle = G^{(2)}(0) \propto \langle \psi | \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} | \psi \rangle = \sum_n |\langle n | \hat{a}\hat{a} | \psi \rangle|^2 \approx \sum_{n=0}^{\infty} (n+1)(n+2) P_{n+2}$$

← probability of n+2 photons in mode

← Two paths ("Bose enhancement")

For short counting time, with $\langle \hat{n} \rangle \ll 1$, $G^{(2)}(0) \approx 2p_2$: probability of 2-photons in mode.

Thermal state: $P_n = \frac{\langle \hat{n} \rangle^n}{(1+\langle \hat{n} \rangle)^{n+1}} \Rightarrow P_2 = \frac{1}{(1+\langle \hat{n} \rangle)^3} \langle \hat{n} \rangle^2 \approx \langle \hat{n} \rangle^2$

Coherent State: $P_n = \frac{1}{n!} \langle \hat{n} \rangle^n e^{-\langle \hat{n} \rangle} \Rightarrow P_2 = \frac{1}{2} \langle \hat{n} \rangle^2 e^{-\langle \hat{n} \rangle} \approx \frac{1}{2} \langle \hat{n} \rangle^2$

The probability of finding two photons in a mode of a thermal state is twice that of the coherent state for the same mean photon number $\langle \hat{n} \rangle$. The photons are thus twice as likely to arrive at the same time — the photons in the thermal state are bunched.

This bunching can be interpreted in terms of the differing photon statistics of these states. The deviation of the coincident counts from "accidental" product of uncorrelated counts

$$\langle \Delta \hat{I}^2 \rangle = \langle : \hat{I}(0)\hat{I}(0): \rangle - \langle : \hat{I}(0): \rangle \langle : \hat{I}(0): \rangle = \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2$$

$$= \langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2 = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 - \langle \hat{n} \rangle = \Delta n^2 - \langle \hat{n} \rangle$$

For a thermal state: $\Delta n^2 = \langle \hat{n} \rangle^2 + \langle \hat{n} \rangle$ (Bose-Einstein dist.) $\Rightarrow \langle \Delta \hat{I}^2 \rangle = \langle \hat{n}^2 \rangle$

For a coherent state: $\Delta n^2 = \langle \hat{n} \rangle$ (Poisson) $\Rightarrow \langle \Delta \hat{I}^2 \rangle = 0$

Classical vs. Nonclassical Light

The photon statistics of the field is one way for us to distinguish "classical light" vs. "non classical" light. We have seen that a classical deterministic current leads to the creation of a coherent state, the quasiclassical state. A classically noisy current leads to a statical mixture of coherent states with a positive $P(\{\alpha_k\})$ - representation.

> Thus, we can define classical light as those with state $\hat{\rho} = \int d\{\alpha_k\}\, P(\{\alpha_k\})\, |\{\alpha_k\}\rangle\langle\{\alpha_k\}|$ where $P(\{\alpha_k\}) \geq 0$. Otherwise, we call the light "nonclassical"

Poisson Statistics

As we have seen, measuring the intensity fluctuations in a mode, equivalent to measuring the difference between photon correlations and the product of uncorrelated "accidentals" at a given time is:

$$\langle \Delta \hat{I}^2 \rangle = \langle : \hat{I}(0)^2 : \rangle - \langle : \hat{I}(0) : \rangle \langle : \hat{I}(0) : \rangle = \Delta n^2 - \langle \hat{n} \rangle \quad : \text{(Deviation from Poisson)}$$

For classical light: $\langle : \hat{I}(0)^2 : \rangle = \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle = \mathrm{Tr}\left(\hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \int d^2\alpha\, P(\alpha)\, |\alpha\rangle\langle\alpha| \right) = \int d^2\alpha\, P(\alpha)\, \langle\alpha| \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} |\alpha\rangle$

$$= \int d^2\alpha\, |\alpha|^4\, P(\alpha) = \overline{|\alpha|^4}$$

$$\langle : \hat{I}(0) : \rangle = \langle \hat{a}^\dagger \hat{a} \rangle = \int d^2\alpha\, |\alpha|^2\, P(\alpha) = \overline{|\alpha|^2}$$

$$\langle \Delta \hat{I}^2 \rangle = \overline{|\alpha|^4} - \left(\overline{|\alpha|^2}\right)^2 = \left(\Delta |\alpha|^2\right)^2 \geq 0$$

$$\Rightarrow \Delta n^2 = \underbrace{\langle \hat{n} \rangle}_{\text{Particle uncertainty}} + \underbrace{\left(\Delta |\alpha|^2\right)^2}_{\text{Wave uncertainty}} \geq \langle \hat{n} \rangle \quad : \text{Super-Poissonian Number Fluctuations}$$

> - For classical light $\Delta n^2 \geq \langle \hat{n} \rangle$ (Super-Poissonian)
> - Sub-Poissonian number statistics $\Delta n^2 < \langle \hat{n} \rangle$ is a signature of non-classical light.

Photon antibunching

Consider the two-time intensity correlation function

$$G^{(2)}(\tau) = \langle : \hat{I}(0) \hat{I}(\tau) : \rangle = \int d[\mathcal{E}]\, P[\mathcal{E}]\, |\mathcal{E}(0)|^2\, |\mathcal{E}(\tau)|^2 = \left(|\mathcal{E}(0)|^2 \middle| |\mathcal{E}(\tau)|^2 \right) \quad \text{(inner product of function)}$$

Cauchy-Schwartz inequality: $\left(|\mathcal{E}(0)|^2 \middle| |\mathcal{E}(\tau)|^2 \right) \leq \sqrt{\left(|\mathcal{E}(0)|^2 \middle| |\mathcal{E}(0)|^2 \right)} \sqrt{\left(|\mathcal{E}(\tau)|^2 \middle| |\mathcal{E}(\tau)|^2 \right)} = \sqrt{\langle : \hat{I}^2(0) : \rangle} \sqrt{\langle : \hat{I}^2(\tau) : \rangle} = \langle : \hat{I}^2(0) : \rangle$

stationary statistics

$$\Rightarrow \langle : \hat{I}(0)\hat{I}(\tau) : \rangle = G^{(2)}(\tau) \leq \langle : \hat{I}(0)^2 : \rangle = G^{(2)}(0)$$

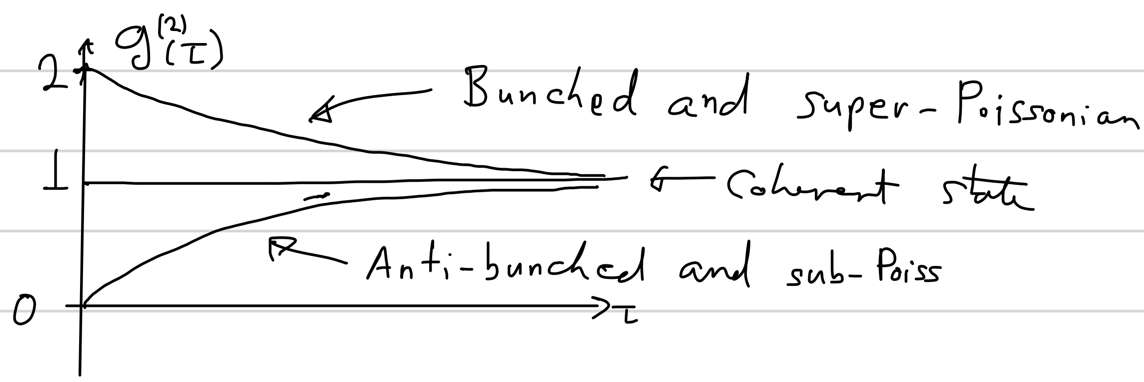
Normalized: $g^{(2)}(0) \geq g^{(2)}(\tau)$: Photon Bunching (classical light)

Nonclassical light $g^{(2)}(0) \leq g^{(2)}(\tau)$:

$\Rightarrow$ Rate of coincidence at zero delay $\leq$ Rate of coincidence at finite delay.

Photons are more likely to arrive separately than together

Nonclassical light $\Rightarrow$ Photon Antibunching



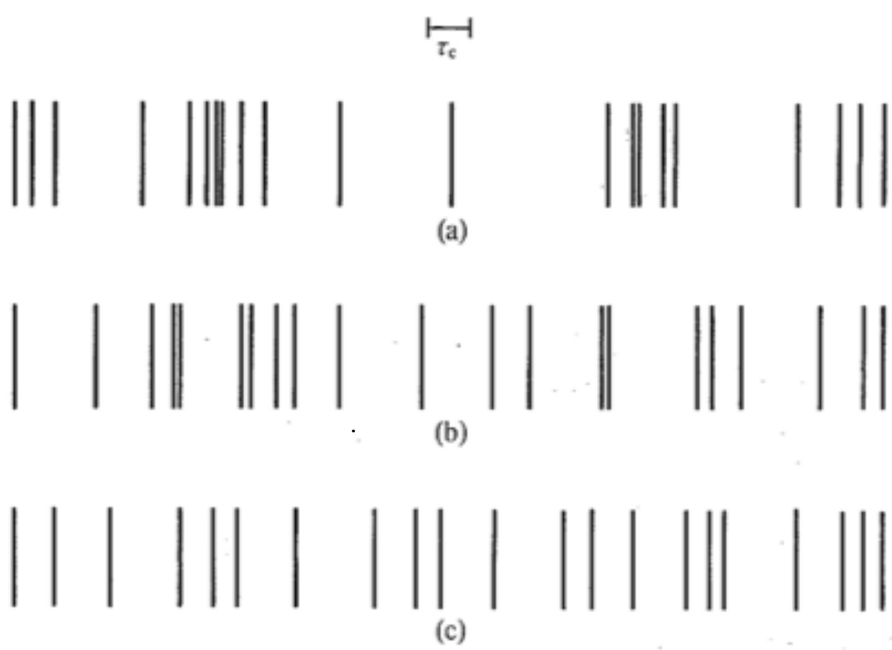


FIG. 6.4. Schematic representations of photon counts as functions of the time for light beams that are (a) bunched with $g^{(2)}(0) > 1$, (b) random with $g^{(2)}(0) = 1$, and (c) antibunched with $g^{(2)}(0) < 1$.

Taken from Loudon, "The Quantum Theory of Light"

The observation of photon anti-bunching and sub-Poissonian photon statistics was a major milestone in the study of nonclassical light.