Physics 566: Quantum Optics I
Problem Set 5
Due Thursday, October 5, 2017

Problem 1: Adiabatic rapid passage (10 Points)

Suppose we have an *inhomogeneously* broadened system - e.g. a system of two-level oscillators with a distribution of resonance energies such as a thermal gas (Doppler broadening) or with a distribution is a solid crystal due to local strain effects. How can we apply a π-pulse to send all of atoms to the excited state with high probability?

(a) Qualitatively suppose we apply a radiation field with a frequency well below resonance ($\Delta \ll \Omega$) and sweep the field slowly up through resonance, ending well above resonance ($\Delta \gg \Omega$), on a time scale much slower that the Rabi frequency $T \gg \Omega^{-1}$, but fast compared to spontaneous emission $T \ll \Gamma^{-1}$. Use the Bloch-sphere magnetic resonance picture to show that population in the ground state will "adiabatically" be transferred to the excited state.

(b) Sketch the eigenvalues of the two-level atom in the RWA as a function of the frequency of the laser. Use the adiabatic theorem of quantum mechanics to explain the transfer of population from the ground to excited state, and the constraints on the time scales. What is the condition that we can invert the entire inhomogeneously broadened sample.

Problem 2: Light forces on atoms (20 Points)

Electromagnetic fields can exert forces on atoms. This is force can be dissipative (the basis of laser cooling) or conservative (the basis for optical trapping, such as optical lattices). Suppose we are given a monochromatic, uniformly polarized laser field of the form $\mathbf{E}(\mathbf{x},t) = \vec{\varepsilon}_L E_0(\mathbf{x})\cos(\omega_L t + \phi(\mathbf{x}))$. The interaction of this field with a two-level atom is described by the Hamiltonian in the rotating frame,

$$\hat{H}_{AL}(\mathbf{R}) = \frac{\hbar\Omega(\mathbf{R})}{2}\left(e^{-i\phi(\mathbf{R})}|e\rangle\langle g| + e^{i\phi(\mathbf{R})}|g\rangle\langle e|\right),$$

where $\mathbf{R}$ is the center of mass position of the atom, and $\hbar\Omega(\mathbf{R}) = -\langle e|\hat{\mathbf{d}}\cdot\vec{\varepsilon}_L|g\rangle E_0(\mathbf{R})$.

Assuming the internal state of the atom relaxes to its steady state much faster than the atom moves, we can neglect the quantum mechanics of the atom's center of mass, and treat its motion as a classical point particle (this is know as the "semiclassical model"). The force on the atom is the defined by the expectation value

$$\mathbf{F} = -\left\langle \nabla \hat{H}_{AL}(\mathbf{R}) \right\rangle = -Tr\left(\nabla \hat{H}_{AL}(\mathbf{R}) \hat{\rho}(t) \right),$$

where $\hat{\rho}(t)$ is the "internal state" of the atoms according to the optical Bloch equations.

(a) Under these condition show that the mean force on the atom is, $\mathbf{F} = \mathbf{F}_{diss} + \mathbf{F}_{react}$, where

$\mathbf{F}_{diss} = -\frac{1}{2}\hbar \mathrm{v(t)}\, \Omega(\mathbf{R}) \nabla \phi(\mathbf{R})$ is the "dissipative force" and

$\mathbf{F}_{react} = -\frac{1}{2}\hbar u(t) \nabla \Omega(\mathbf{R})$ is the "reactive force",

with u and v the components of the Bloch vector in the rotating frame relative to the incident phase $\rho_{ge} e^{i\phi(\mathbf{R})} = (u + i\mathrm{v})/2$.

(b) Show that in steady state, the rate at which that laser does work on the atom, averaged over an optical period is:

$\left\langle \frac{dW}{dt} \right\rangle_{s.s} = \frac{\hbar \Omega_0 \omega_L}{2} \mathrm{v}_{\mathrm{s.s}} = \gamma_s \hbar \omega_L$, where $\gamma_s = \Gamma \rho_{ee}^{s.s.}$ is the photon scattering rate.

Interpret this result.

(c) For the case of a plane wave $\mathbf{E}(\mathbf{R},t) = \vec{\varepsilon}_L E_0 \cos(\omega_L t - \mathbf{k} \cdot \mathbf{R})$, show that in steady-state:

$\mathbf{F}_{diss} = \gamma_s \hbar \mathbf{k}_L$. This is known as "radiation pressure" or the "scattering force" - interpret.

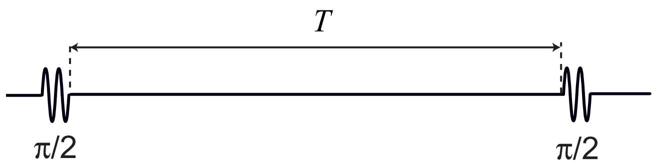
(d) For the "reactive force" consider the case of weak saturation, s<<1. Show that

$$\mathbf{F}_{react} = -\nabla U(\mathbf{R}),$$

where the optical "dipole force" is $U(\mathbf{R}) = \hbar \Delta(\mathbf{R}) \frac{s}{2} = -\frac{1}{4} \mathrm{Re}(\tilde{\alpha}) \left| E_0(\mathbf{R}) \right|^2$ -- interpreted the physical meaning of $U(\mathbf{R})$.

Problem 3: Ramsey fringes and the measurement of T2 times (20 Points – Extra Credit)

(a) We seek to measure the coherence of between the computational basis states of a qubit $\{|0\rangle,|1\rangle\}$. Consider a two-pulse Ramsey sequence: A "hard" π/2 pulse around *x*-axis with detuning Δ, free evolution for a time *T*, a second hard π/2 pulse around *x*-axis at the same detuning Δ.



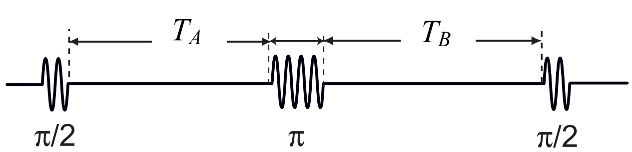
During the free evolution, the coherence ρ_{01} decays exponentially with rate $1/T_2$. Show that, given the qubit initially in $|1\rangle$, the probability to find $|0\rangle$ after the sequence is

$$P_0 = \frac{1}{2}\left[1+\cos(\Delta T)e^{-T/T_2}\right]$$

Explain this using the evolution on the Bloch sphere. Plot this for $\Delta/2\pi$ = 1 MHz and T_2=25 μs, for T=0 to 25 μs.

(b) Suppose now that in addition to homogeneous decay, there is inhomogeneous decay T_2^*. Suppose that if the pulses are tuned to frequency ω, the probability the detuning seen by the qubit is Gaussian distributed, $p(\Delta)=e^{-\frac{(\Delta-\Delta_0)^2}{2\delta^2}}/\sqrt{2\pi\delta^2}$, where Δ_0 is the mean detuning and $\delta = 1/T^*{}_2$ is the spread in detunings. Calculate the probability P_0 in the same two-pulse Ramsey sequence of part (a) for $T_2^* = 5$ μs. Comment on the result.

(c) Now consider a three-pulse Hahn spin-echo Ramsey sequence:
A "hard" π/2 pulse around *y*-axis with detuning Δ, free evolution for a time T_A, a "time reverse" hard π pulse around *x*, free evolution for a time T_B, and then a second hard π/2 pulse around *y*-axis at the same detuning Δ.



Show, $P_0 = \frac{1}{2}\left(1-\cos\left[\Delta(T_A-T_B)\right]e^{-\delta^2(T_A-T_B)^2/2}e^{-(T_A+T_B)/T_2}\right)$, and plot for $T_A = 10\mu s$, as a function of T_B=0 to 25 μs.