Physics 566: Quantum Optics I

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Lecture 1: Introduction

Quantum Optics, i.e. the quantum theory of light (EM radiation) was at the center of the birth of quantum mechanics

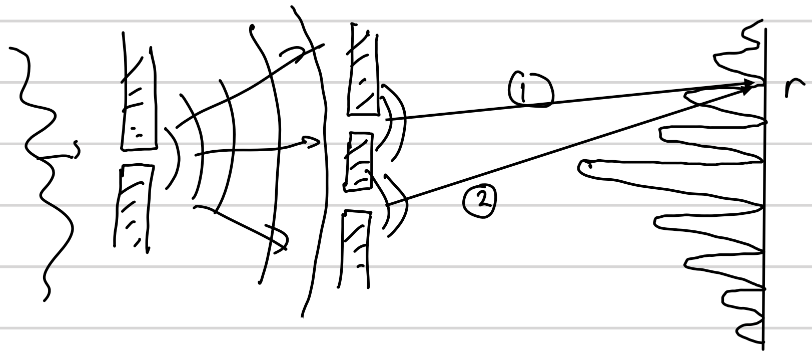
- 1900: Planck black-body radiation law.

  Introduces the "quantum of action"

- 1905: Einstein introduces "quantum of light", "photon"
  - Explanation of Plank law
  - Photoelectric effect

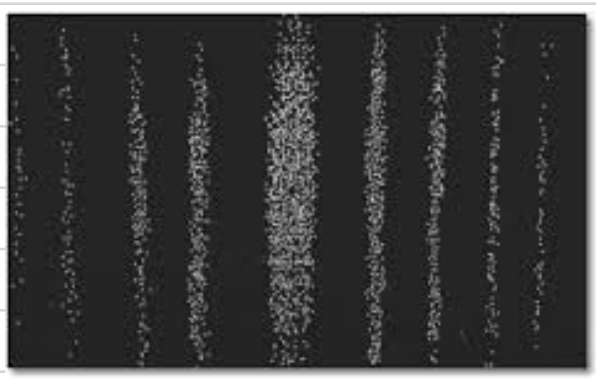
Wave particle duality becomes of cornerstone of the new quantum theory

- 1803: Young's double slit experiment had "unambiguously" proven that light was a wave



Interference of electric field $E(r) = E_1 + E_2 \Rightarrow$ Intensity $I(r) \propto (E_1 + E_2)^2$

- 1909: Thompson-Taylor: Interference of "feeble light" at the level of single photon. Interference pattern builds up photon-by-photon



Light has a "corpuscular," "granular" nature

New notion of interference
$\Rightarrow$ Probability amplitude

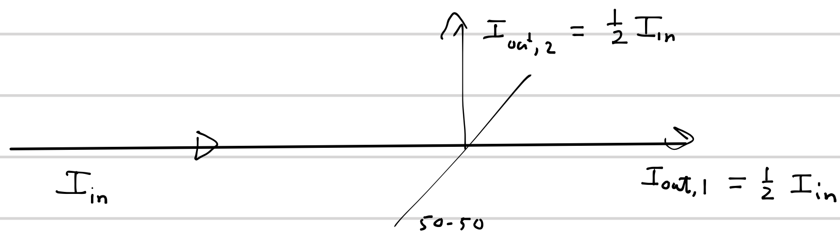
Feynman: "The double-slit experiment has in it the heart of quantum mechanics. Really, it contains the only mystery."

~1923 Compton effect: Inelastic scattering of an x-ray by a photon → cannot be explained by scattering of waves → light has corpuscular properties → the photon!

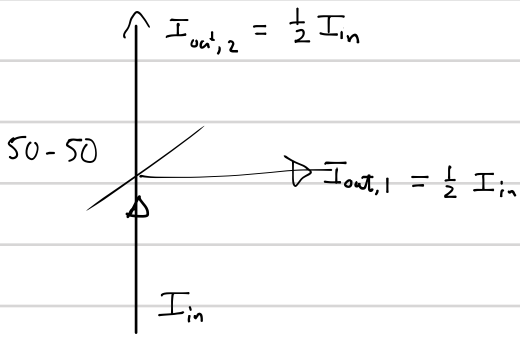
• Quantum Optics: The study, manipulation, and control of quantum mechanical coherence associated with optical (electromagnetic) fields.

• Coherence: The capacity of a system to exhibit interference

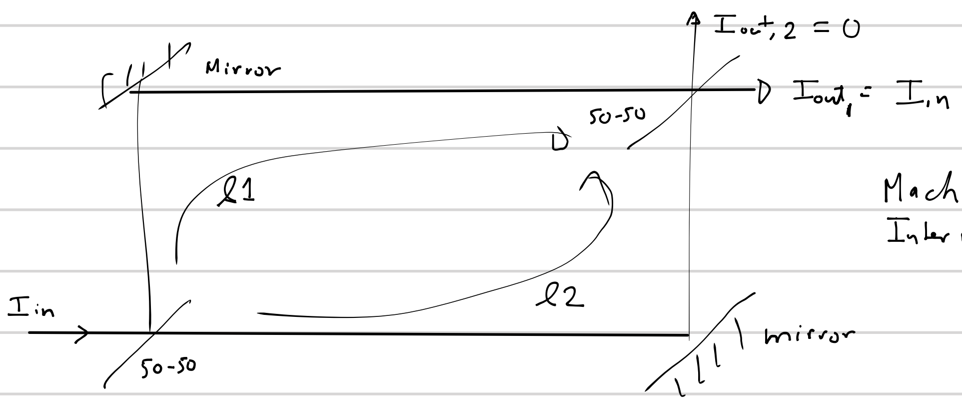
Let us begin by recalling the interference of classical electromagnetic waves. A light beam, approximated as a plane wave, is incident on a 50-50 beam splitter



50-50 means 50% of the intensity is transmitted and 50% is reflected. Of course, the same would be true if the input field were incident in the other input port.



But if we redirected these two beams to another beam splitter, something surprising happens



Mach-Zender Interferometer

As long as the two paths $l1$ and $l2$ are equal, all of the intensity exits port-1 of the second 50-50 beam splitter, and no intensity exist port 2?
This, of course is interference – the two alternative paths are said to interfere such that there is constructive interference for the beam to emerge port-1 and complete destructive interference for emerging port-2.

Interference is a wave phenomena. The intensity (energy flux) is related to the square of the wave amplitude. For electromagnetic waves, $I = |\langle \vec{S} \rangle| = \frac{c}{4\pi} \langle \vec{E} \times \vec{B} \rangle$ (Time average of Poynting vector)

For plane wave in vacuum, propagating in the z-direction. $\vec{E} = \mathrm{Re}\left[\underset{\sim}{E_0}\, e^{i(kz-\omega t)}\right]$

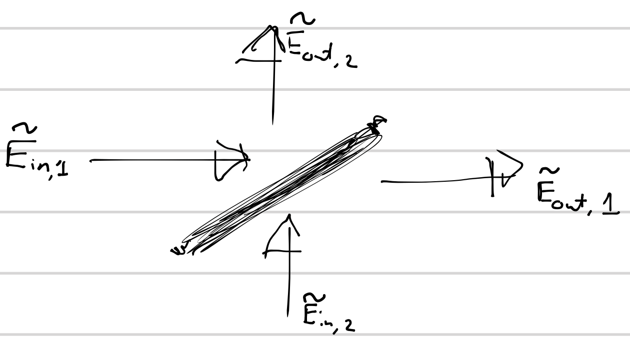
$$\vec{B} = \vec{e}_z \times \vec{E}$$

$$I = \frac{c}{8\pi} |\underset{\sim}{\vec{E}_0} e^{ikz}|^2 = \frac{c}{8\pi} |\underset{\sim}{\vec{E}_0}|^2$$ (we will drop the factor $\frac{c}{8\pi}$ when we don't need it)

Maxwell's equations are linear in $\vec{E}$ & $\vec{B}$ (for prescribed sources), so the principle of superposition holds for fields, fields add, not intensities. Thus, given a total field $\vec{E}_{total} = \vec{E}_1 + \vec{E}_2$ (complex amplitudes), the total intensity

$$I_{total} = |\underset{\sim}{\vec{E}_1} + \underset{\sim}{\vec{E}_2}|^2 = \underbrace{|\underset{\sim}{\vec{E}_1}|^2}_{I_1} + \underbrace{|\underset{\sim}{\vec{E}_2}|^2}_{I_2} + \underbrace{2\,\mathrm{Re}(\underset{\sim}{\vec{E}_1} \cdot \underset{\sim}{\vec{E}_2^*})}_{\text{Interference?}}$$

Example: Symmetric beamsplitter



Amplitude reflection / transmission
r / t

Scattering of waves

$$\begin{bmatrix} \tilde{E}_{out,1} \\ \tilde{E}_{out,2} \end{bmatrix} = \begin{bmatrix} t & r \\ r & t \end{bmatrix} \begin{bmatrix} \tilde{E}_{in,1} \\ \tilde{E}_{in,2} \end{bmatrix} \Rightarrow \begin{aligned} \tilde{E}_{out,1} &= t\,\tilde{E}_{in,1} + r\,\tilde{E}_{in,2} \\ \tilde{E}_{out,2} &= t\,\tilde{E}_{in,2} + r\,\tilde{E}_{in,1} \end{aligned}$$

$$I_{out,1} = |\tilde{E}_{out,1}|^2 = |t|^2 I_{in,1} + |r|^2 I_{in,2} + tr^* \tilde{E}_{in,1} \tilde{E}^*_{in,2} + t^* r\, \tilde{E}^*_{in,1} \tilde{E}_{in,2}$$

$$I_{out,2} = |E_{out,2}|^2 = |t|^2 I_{in,2} + |r|^2 I_{in,1} + tr^* \tilde{E}^*_{in,1} \tilde{E}_{in,2} + t^* r\, \tilde{E}_{in,1} \tilde{E}^*_{in,2}$$

Assuming lossless beam splitters, because energy is conserved,

$$I_{out,1} + I_{out,2} = (|r|^2 + |t|^2)(I_{in,1} + I_{in,2}) + (rt^* + tr^*)(\tilde{E}^*_{in,1}\tilde{E}_{in,2} + \tilde{E}_{in,1}\tilde{E}^*_{in,2})$$
$$= I_{in,1} + I_{in,2}$$

$\Rightarrow |r|^2 + |t|^2 = 1$, $rt^* + t^*r = 0$ $\Rightarrow$ $\begin{bmatrix} t & r \\ r & t \end{bmatrix}$ is a unitary matrix, $U(2)$

Note, if we write $r = \sqrt{R}\, e^{i\phi_r}$, $t = \sqrt{T}\, e^{i\phi_t}$

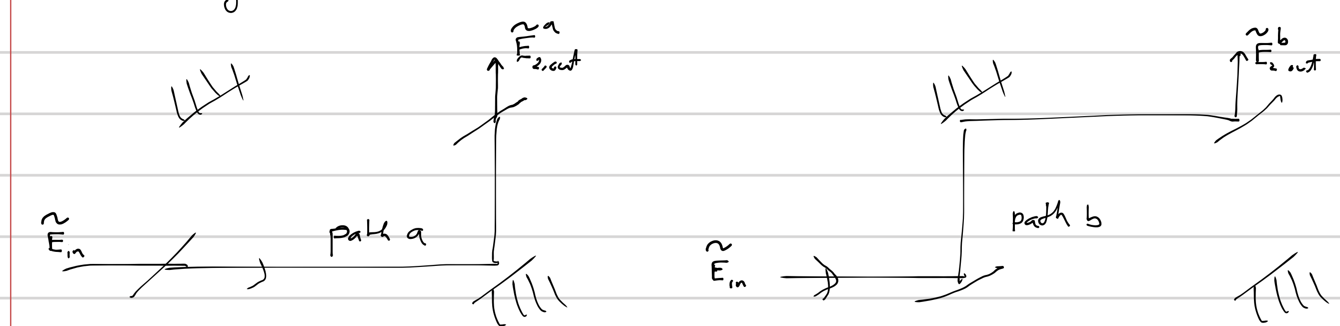
$$R + T = 1, \quad \phi_t = \phi_r \pm \pi/2$$

For a symmetric 50-50 beam splitter, $\begin{bmatrix} t & r \\ r & t \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$ Note $\det\begin{bmatrix} t & r \\ r & t \end{bmatrix} = 1$

$\Rightarrow SU(2)$

With this in combination with mirrors, one can show that the balanced Mach-Zender interferometer yield destructive interference for port-2.

Two interfering paths



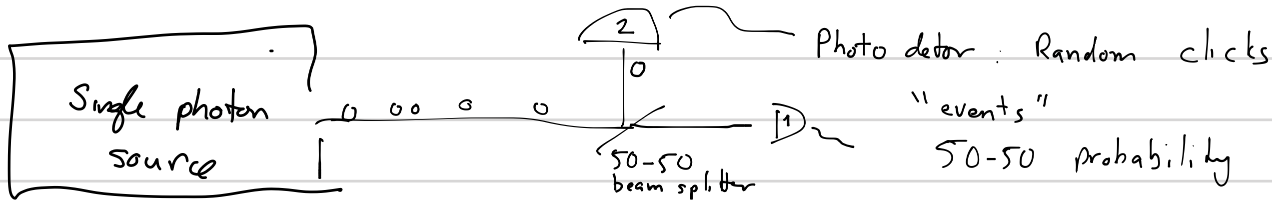
$$\tilde{E}_{2,out} = \tilde{E}^a_{2,out} + \tilde{E}^b_{2,out} = (t)(-1)(t)e^{ikl_a}\tilde{E}_{in} + (r)(-1)(r)e^{ikl_b}\tilde{E}_{in}$$

beam-splitter, mirror, beam splitter, propagation phase shift

For a balanced Mach-Zender $l_a = l_b$. 50-50 beam splitter: $r = \frac{i}{\sqrt{2}}$, $t = \frac{1}{\sqrt{2}}$

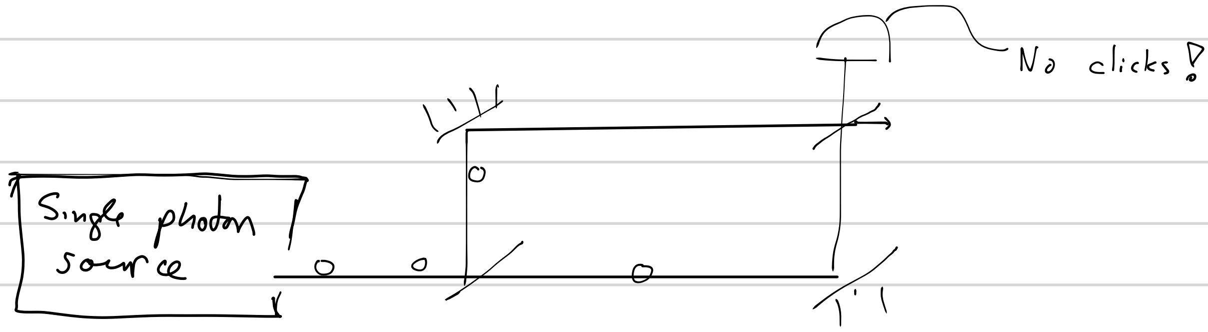
$\Rightarrow \tilde{E}_{2,out} = -\frac{1}{2}e^{ikl}\tilde{E}_{in} + \frac{1}{\sqrt{2}}e^{ikl}\tilde{E}_{in} = 0 \Rightarrow$ Destructive interference

Quantum Coherence: Wave/Particle duality and beyond

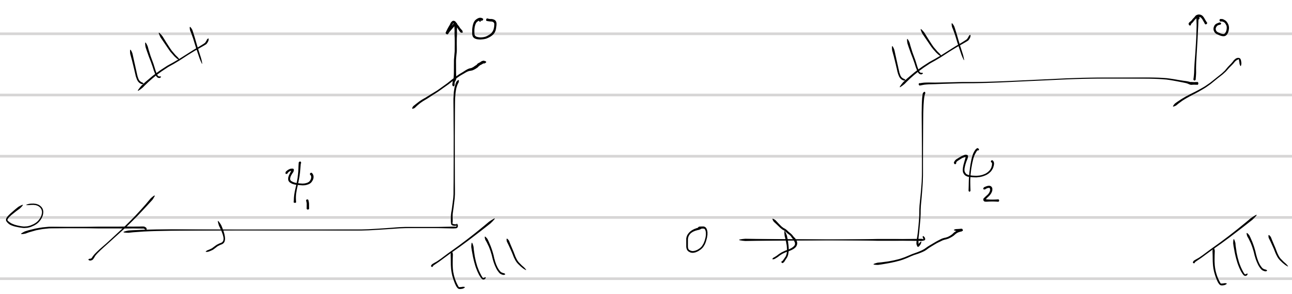


Let us consider a quantum optical source which delivers a stream of individual photons. Each photon has a probability of being reflected and transmitted at the beam splitter. Whereas the classical intensity is always divided, the photons are quanta that are indivisible; either detector-1 or detector-2 clicks.

When there are alternative processes that indistinguisably lead to the same event, they interfere. This is quantum coherence.



Interference of processes (photon by photon): Probability amplitudes!

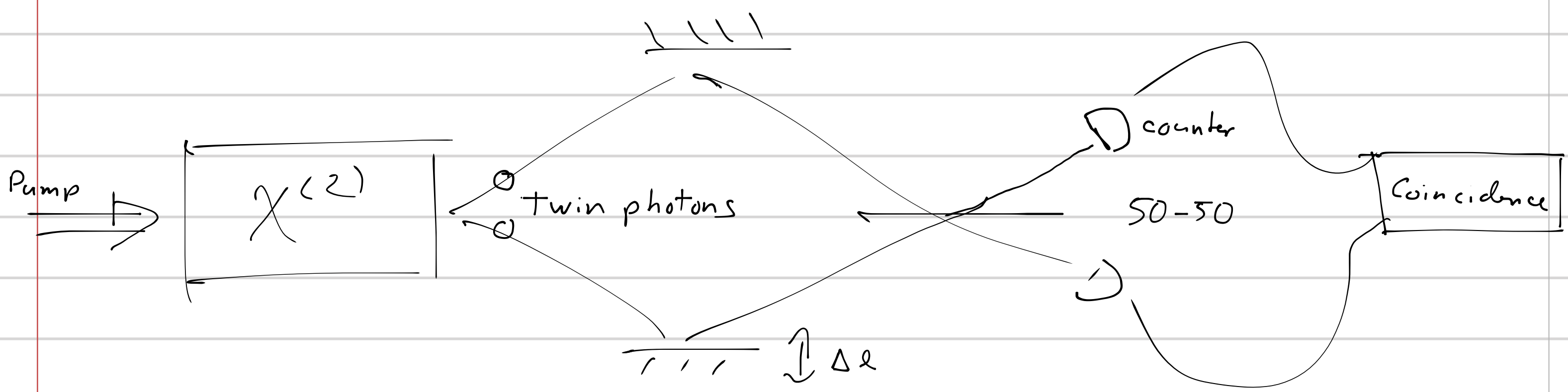


$$\psi_1 + \psi_2 = 0$$

The classical wave interference we see at the macroscopic level can be reinterpreted as interference of the probability amplitudes for different processes of the particles. The intensity we observe is the average over many discrete events.
While there is a quantum aspect to this Mach–Zehnder interferometer, probability of events is equivalent to classical wave interference. An important goal in quantum optics is to understand the electromagnetic fields that are essentially nonclassical.

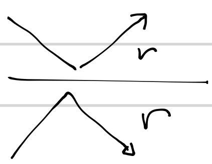
Quantum Coherence: Interference of indistinguishable processes

Beyond classical optics: Example - Two-photon interference

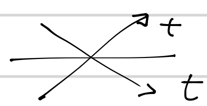


Pairs of identical (twin) photons are incident on a 50-50 beam splitter
We seek those events in which both detectors go "click" simultaneously

There are two indistinguishable processes that lead to these events



Both are reflected
$\psi_r = r^2$

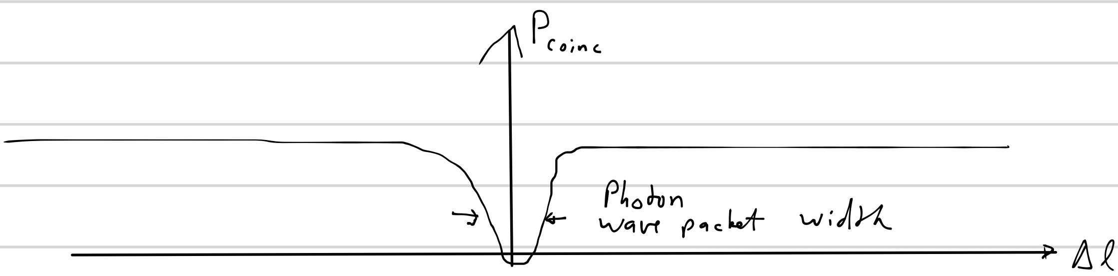


Both are transmitted
$\psi_t = t^2$

The total probability for seeing coincidence is the sum of the amplitudes, and then squared

$$P_{coincidence} = |\psi_r + \psi_t|^2 = |r^2 + t^2|^2 = \left|\left(\tfrac{i}{\sqrt{2}}\right)^2 + \left(\tfrac{1}{\sqrt{2}}\right)^2\right|^2 = 0 \;!$$

The two processes destructively interfere



This is the famous "Hong-Ou-Mandel" dip.

Quantum Superpositions and Interference

The Born-rule tells us that if the probilities amplitude for a certain event is α, the the probability of seeing that event in a measurement is $P = |\alpha|^2$.

From the general structure of Quantum Mechanics, a measurement outcome is

- typically represented by an eigenstate $|a\rangle$ of some observable $\hat{A}$
- The state of the system is represented by a vector $|\psi\rangle$ in the Hilbert space

(We will generalize this later, beyond "pure states" and "projective measurements")

The Born rule then reads, $P(a|\psi) = |\langle a|\psi\rangle|^2$

Now suppose the state of the system is in a superposition of some states,

E.g $|\psi\rangle = \alpha|\phi_\alpha\rangle + \beta|\phi_\beta\rangle$

$$\Rightarrow P(a|\psi) = |\alpha\langle a|\phi_\alpha\rangle + \beta\langle a|\phi_\beta\rangle|^2$$

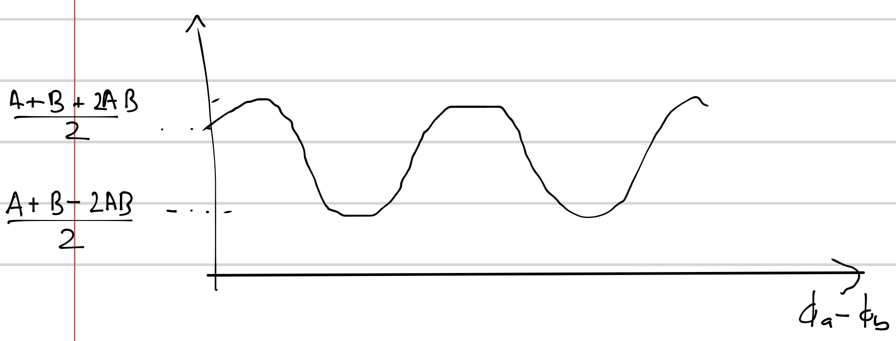
$$= |\alpha|^2 |\langle a|\phi_\alpha\rangle|^2 + |\beta|^2 |\langle a|\phi_\beta\rangle|^2 + \alpha\beta^* \langle a|\phi_\alpha\rangle\langle\phi_\beta|a\rangle + \alpha^*\beta\langle a|\phi_\beta\rangle\langle\phi_\beta|a\rangle$$

$$= \underbrace{P(a|\phi_\alpha)P(\phi_\alpha) + P(a|\phi_\beta)P(\phi_\beta)}_{\text{Logical? Conditional probability}} + \text{Interference term}$$

Everything that's strange and wonderful about quantum mechanics

The interference terms represent quantum coherence — phase relationship between amplitudes α + β. Suppose $\langle a|\phi_\alpha\rangle = \langle a|\phi_\beta\rangle = \frac{1}{\sqrt{2}}$, $\alpha = \sqrt{A}e^{i\phi_a}$, $\beta = \sqrt{B}\,e^{i\phi_B}$

$$\Rightarrow P(a|\psi) = \tfrac{1}{2}A + \tfrac{1}{2}B + AB\cos(\phi_a - \phi_B)$$

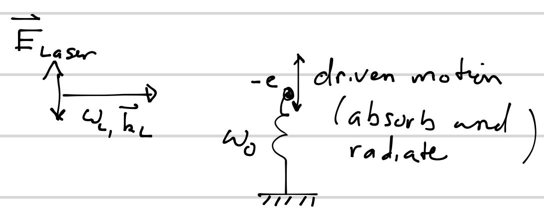


Fringe visibility : $\dfrac{\text{Max} - \text{Min}}{\text{Max} + \text{Min}}$

$= \dfrac{2AB}{A+B}$: Measures degree of coherence

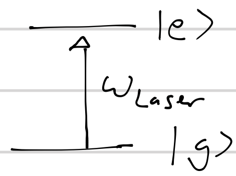
Example: Atomic coherence (Dark State)

Classically, a bound charge will resonantly absorb/emitted electromagnetic radiation



Quantum mechanically, we can resonantly absorb when the laser frequency matches the energy difference between two energy levels. For example, in atom we have a well defined ground state and excited energy levels:

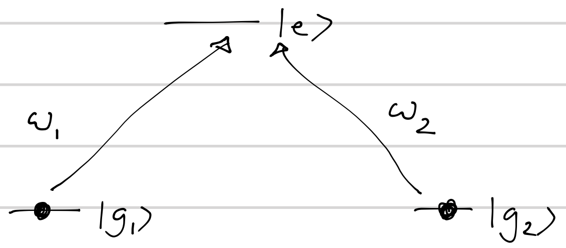
Absorption in an atomic resonance



Two-level atom

The atomic response can be coherent, i.e., lead to quantum superposition of levels $|e\rangle$ and $|g\rangle$ $\Rightarrow$ Nonclassical atom response

Three-level atom (lambda configuration)



Two possible "paths" to absorption. There exists a superposition $|\psi_{dark}\rangle = \alpha_1 |g_1\rangle + \alpha_2 |g_2\rangle$, such that these two paths destructively interfere.

Atoms in this state are said to be dark because the don't absorb (or emit) light.

Another important goal in quantum optics is to understand the quantum-coherent properties of matter.