Physics 566: Quantum Optics I
Problem Set #9
Due Tuesday, November 11, 2025

Problem 1: Thermal Light (25 points)

Consider a single mode field in thermal equilibrium at temperature T, Boltzmann factor $\beta = 1/k_B T$. The state of the field is described by the "canonical ensemble",

$\hat{\rho} = \frac{1}{Z} e^{-\beta \hat{H}}$, $\hat{H} = \hbar\omega \hat{a}^\dagger \hat{a}$ is the Hamiltonian and $Z = Tr\left(e^{-\beta \hat{H}}\right)$ is the partition function.

(a) Remind yourself of the basic properties by deriving the following:

- $\langle n \rangle = \dfrac{1}{e^{\beta\hbar\omega} - 1}$ (the Planck spectrum)
- $P_n = \dfrac{\langle n \rangle^n}{\left(1 + \langle n \rangle\right)^{n+1}}$ (the Bose-Einstein distribution).

(b) Make a bar-plot of P_n for both the thermal state and the coherent state on the same graph as a function of *n,* for each of the following: $\langle n \rangle = 0.1, 1, 10, 100$.

(c) Use the Bose-Einstein distribution to show that for a thermal state

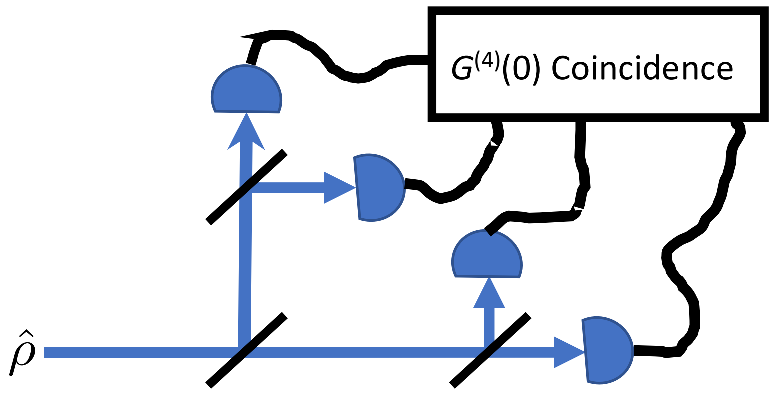
$$\Delta n^2 = \langle n \rangle^2 + \langle n \rangle$$

(d) Show that the Glauber-Sudharshan distribution of this state, $P(\alpha) = \dfrac{1}{\pi \langle n \rangle} e^{-|\alpha|^2/\langle n \rangle}$, satisfies

$$\int d^2\alpha\, P(\alpha) |\alpha\rangle\langle\alpha| = \sum_n \frac{\langle n \rangle^n}{\left(1 + \langle n \rangle\right)^{n+1}} |n\rangle\langle n| .$$

Problem 2: Bose statistics and photon correlations – number basis (25 points).

Consider the coincidence count of *m*-photons in a given temporal mode $G^{(m)}(0) = \langle : \hat{n}^m : \rangle$. For example, the measurement of the four-point correlation function is depicted below.



a) Suppose the input state an *n*-photon Fock state. Show that the m$^{\text{th}}$ order Glauber correlation is

$$G^{(m)}(0) = \langle n| : \hat{n}^m : |n\rangle = m! \binom{n}{m} = \frac{n!}{(n-m)!}.$$

Interpret this in terms of m-particle interference of identical bosons.

b) For a general state show that

$$G^{(m)}(0) = \sum_n P_n \frac{n!}{(n-m)!}, \quad \text{where } P_n = \langle n|\hat{\rho}|n\rangle$$

c) For a coherent state, $P_n = \frac{\langle \hat{n} \rangle^n}{n!} e^{-\langle \hat{n} \rangle}$, use b) to show that $G^{(m)}(0) = \langle \hat{n} \rangle^m$.

d) For a thermal state, $P_n = \frac{\langle \hat{n} \rangle^n}{(1+\langle \hat{n} \rangle)^{n+1}}$, use b) to show that $G^{(m)}(0) = m! \langle \hat{n} \rangle^m$.

e) Repeat (d) and confirm you get the same result using the Glauber-P representation. Interpret in terms of intensity fluctuations in "chaotic light."