Optical properties of electromagnetically induced transparency

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A review of electromagnetically induced transparency is given. A semiclassical Hamiltonian is used to show that atomic coherence can inhibit absorption and enhance dispersion in the adiabatic limit. Experimental demonstrations and applications are discussed. The nonadiabatic effects are derived in the c-number Langevin formalism, and shown to give rise to correlate the phase of two incident fields.

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I. INTRODUCTION

Electromagnetically induced transparency (EIT) [1] occurs when a coherent superposition of atomic states inhibits optical transitions via destructive interference. This is induced in a three level Λ-type atomic system (Fig. 1) by applying an electromagnetic field to each single photon transition. Under these conditions a medium which is strongly absorptive in the presence of one field can become completely transparent when the second field is applied. This behavior is accompanied by rapid variations in the index of refraction, creating the novel situation of strong dispersion with little or no absorption. These unique properties have a wide range of applications in quantum optics, including subrecoil atomic cooling [2], adiabatic population transfer [3], lasing without inversion (LWI) [1], ultraslow light propagation [4], pulse matching and phase correlation [5], and the enhancement of nonlinear processes such as nondegenerate four wave mixing, frequency conversion [6], and two photon absorption [7].

This review is comprised of two sections. In section II a brief review of the semiclassical theory in the adiabatic following approximation is given which demonstrates the atomic coherence and strong dispersion associated with EIT, and a discussion of a demonstrative experiment is given. This allows an understanding of many of the applications listed above, but a detailed analysis of each of these is beyond the scope of this article. In section III the c-number Langevin formalism is used to demonstrate pulse matching and phase correlation which are not seen in the adiabatic theory.

II. THE SEMICLASSICAL APPROACH

A. Dark States and Electromagnetically Induced Transparency

A schematic of the two fields E_1 and E_2 and the three level system is shown in Fig. 1. It is assumed that in the electric dipole approximation E_1 only couples the states $|1\rangle$ and $|3\rangle$, and E_2 only couples the states $|2\rangle$ and $|3\rangle$, as in the case of a ground state with hyperfine splitting. The population decay rates from $|3\rangle$ to $|1\rangle$, and $|3\rangle$ to $|2\rangle$ are γ_1 and γ_2, respectively, and the total decay rate from $|3\rangle$ is written as $\Gamma= \gamma_1 + \gamma_2$. The two lower states are taken to be very long lived, and population decay between them is neglected, but dephasing of coherent superpositions of the ground states is included as the dephasing rate γ_0. This dephasing rate can be attributed to the long but finite lifetimes of the ground states, and to collisions in a gaseous sample. The detunings are defined as $\Delta = (\omega_1 - \omega_{31})$, and $\delta = (\omega_2 - \omega_1) - (\omega_{32} - \omega_{31})$, where ω_i is the frequency of the field E_i, and ω_{jk} is the transition frequency between the states $|j\rangle$ and $|k\rangle$. Thus Δ is recognized as the one photon detuning, and δ is recognized as the Raman detuning. Doppler effects are ignored.

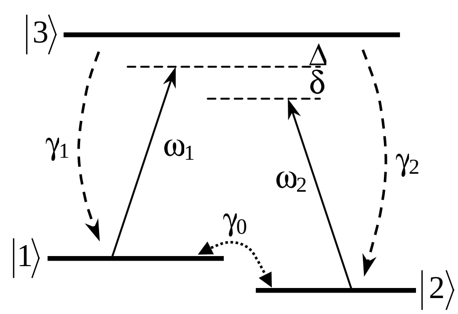

Fig. 1: The Λ-type atomic levels. Δ and δ are the detunings from the one and two photon resonances, respectively.γ_0 is taken to be a pure dephasing rate.

With the basis vectors defined as $|1\rangle = (1,0,0)$, $|2\rangle = (0,1,0)$, and $|3\rangle = (0,0,1)$, the density matrix is transformed to a frame rotating as

$$\hat{\rho}_{13} = \hat{\hat{\rho}}_{13} e^{-i\Delta t}, \quad \hat{\rho}_{23} = \hat{\hat{\rho}}_{23} e^{-i(\Delta+\delta)t}, \quad \hat{\rho}_{12} = \hat{\hat{\rho}}_{12} e^{i\delta t}. \qquad (1)$$

The Hamiltonian in this basis can then be written as

$$\hat{H} = \frac{\hbar}{2}\begin{bmatrix} \delta & 0 & -\Omega_1^* \\ 0 & -\delta & -\Omega_2^* \\ -\Omega_1 & -\Omega_2 & 2(\Delta + \delta/2) \end{bmatrix}, \tag{2}$$

where $\Omega_i \equiv (\wp_{3i} E_i)/\hbar$ are the Rabi frequencies, and $\wp_{3i}$ is the dipole moment. The decay rates are added phenomenologically by writing the relaxation terms as

$$\hat{R} = \begin{bmatrix} \gamma_1 \hat{\rho}_{33} & -\gamma_0 \hat{\tilde{\rho}}_{12} & -(\Gamma/2)\hat{\tilde{\rho}}_{13} \\ -\gamma_0 \hat{\tilde{\rho}}_{21} & \gamma_2 \hat{\rho}_{33} & -(\Gamma/2)\hat{\tilde{\rho}}_{23} \\ -(\Gamma/2)\hat{\tilde{\rho}}_{31} & -(\Gamma/2)\hat{\tilde{\rho}}_{32} & -\Gamma \hat{\rho}_{33} \end{bmatrix}, \tag{3}$$

and adding them to the equations of motion:

$$\dot{\hat{\tilde{\rho}}}_{jk} = \frac{1}{i\hbar}\left[\hat{H}, \hat{\tilde{\rho}}_{jk}\right] + \hat{R}_{jk}. \tag{4}$$

These equations are not analytically solvable, and approximations are required to proceed. But before doing so it is possible to demonstrate the atomic coherence that is at the heart of EIT. To do so we transform to a set of orthogonal basis states defined as $|-\rangle = \cos\theta|1\rangle - \sin\theta|2\rangle$, $|+\rangle = \sin\theta|1\rangle + \cos\theta|2\rangle$, and $|3\rangle = |3\rangle$ under the transformation [8]

$$T = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{5}$$

where $\sin\theta = \Omega_1/\Omega$, $\cos\theta = \Omega_2/\Omega$, and $\Omega = (\Omega_1^2 + \Omega_2^2)^{1/2}$. The Hamiltonian is transformed by $\hat{H}' = T\hat{H}T^{-1}$, giving

$$\hat{H}' = \frac{\hbar}{2}\begin{bmatrix} \delta\cos(2\theta) & \delta\sin(2\theta) & 0 \\ \delta\sin(2\theta) & -\delta\cos(2\theta) & -\Omega \\ 0 & -\Omega & 2(\Delta + \delta/2) \end{bmatrix}. \qquad (6)$$

This Hamiltonian is written in the basis $|-\rangle$ = (1,0,0), $|+\rangle$ = (0,1,0), $|3\rangle$ = (0,0,1). Thus the off-diagonal elements represent the coupling strength between the states, and it can be seen that the $|-\rangle$ state is no longer coupled to the excited state (H_{-3} = 0). This decoupling occurs because the $|1\rangle$ and $|2\rangle$ states are out of phase in this superposition, and the dipole moments coupling these two states to the excited state destructively interfere. This is commonly referred to as the dark state. On the other hand, in the $|+\rangle$ state the two ground states are in phase, so the dipole moments add, and the $|+\rangle$ is strongly coupled to the excited state ($H_{+3} = -\hbar\Omega/2$). This is referred to as the bright state. The dark state is coupled to the bright state by $H_{-+} = (\hbar\delta\sin(2\theta))/2$, and therefore the system will absorb photons unless $H_{-+} = 0$. This occurs when $\Omega_1 = 0$, which corresponds to simple optical pumping via Ω_2 into the $|-\rangle = |1\rangle$ state, or when $\delta = 0$, which corresponds to Raman resonance. This is the condition for EIT. In this case the steady state population will be trapped in the dark state and the absorption of both fields will approach zero.

One interesting feature of the dark state is that the populations of the $|1\rangle$ and $|2\rangle$ in the superposition state are weighted by the field strengths Ω_2 and Ω_1. This means that once EIT is established, one can control the populations of the ground states by slowly changing the intensities of the two fields. Thus it is possible to shift the population from the lower of the two ground states to the upper one. This is referred to as adiabatic population transfer, and is used in both cw and pulsed systems to highly populate an upper ground state which is normally not populated. The population can be made greater than 1/2 (the saturation limit), and can be used as a ground state in another optical process (such as interactions with a fourth level) [9].

Conversely, one can create a dark state with maximal coherence ($|\rho_{12}| = 1/2$). Such a state can then be used as an atomic local oscillator and mixed with a third laser beam to generate light at the sum and difference frequencies. Since the absorption is low for atoms in the dark state the atomic density can be made very high, increasing the efficiency of the mixing process. Furthermore, it can be shown that the third order nonlinear susceptibility of atoms in the dark state is resonantly enhanced, though a derivation of this result lies beyond the scope of this review. These effects were demonstrated by M. Jain et al. [10] in an optically thick ^{208}Pb vapor (a Λ-system), in which the transmission coefficient of the E_2 pulse in the absence of coherence would be $e^{-300000}$. This strong absorption was completely eliminated with EIT. The dark state was generated with maximal coherence with λ_1 = 406 nm, λ_2 = 283 nm, and then a third pulse was applied at 425 nm to generate the sum frequency field at 293 nm with a conversion efficiency of ~40%. Similar techniques were used to enhance nondegenerate four wave mixing with cw fields in an optically thick ^{87}Rb vapor by Yong-qing Li et al. [11].

To demonstrate strong dispersion in the presence of almost zero absorption the first order optical susceptibilities of the EIT feature can be derived from the original equations of motion (Eq. 4) by making the adiabatic following approximation. In this approximation the relaxation of the upper state is taken to be much faster than the time scale over which the populations change: $\gamma_{1,2} >> \Omega_{1,2}$. The population of the upper state ρ_{33} is therefore negligible when compared with that of the lower states ρ_{11} and ρ_{22}, and can be dropped from the equations of motion. The resulting equations are then solved in steady state, giving the absorption and dispersion for the E_i field: $\chi'' \propto \mathrm{Im}(\rho_{3i})$, and $\chi' \propto \mathrm{Re}(\rho_{3i})$. To simplify the final results it is convenient to consider one field to be a strong coupling field, and the other to be a weak probe field, though the choice of field is arbitrary. Keeping terms of lowest order in $\Omega_{1,2}/\gamma_{1,2}$ one finds that for a strong coupling field Ω_1, the relevant matrix element for the propagation of the probe field Ω_2 is

$$\tilde{\rho}_{32} = \frac{\Omega_2}{2(\Delta+\delta-i\Gamma/2)}\left\{1-\frac{\Omega_1^2}{4(\delta-i\gamma_0)(\Delta+\delta-i\Gamma/2)}\right\}^{-1} \quad . \tag{7}$$

The real and imaginary parts of this expression for various strengths of the coupling field and relaxation rates as a function of the Raman detuning are shown in Fig. 2. It can be seen that in the middle of the normal Lorentzian two level absorption profile (centered on the one photon resonance $\Delta+\delta = 0$) is a dip of width $\Delta\omega_{EIT} \approx \gamma_0 + \Omega_1^2/(2\Gamma)$, centered on the Raman resonance $\delta = 0$. Associated with this dip is a strong dispersion as shown in Fig. 2(a). In Fig. 2(b), it can be

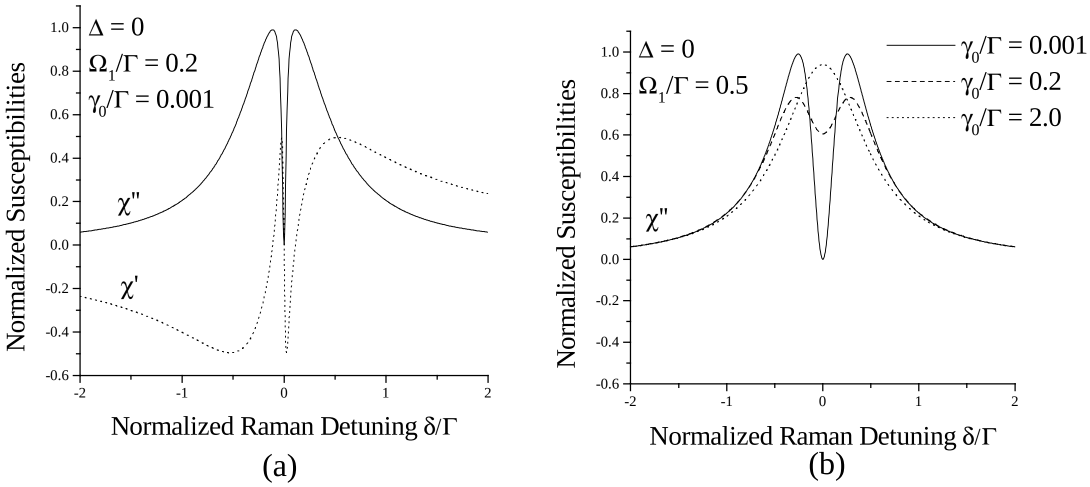

Fig. 2:(a) The real and imaginary susceptibilities showing the strong dispersion and dip in absorption for EIT. (b) The imaginary susceptibility for three different ground state coherence lifetimes γ_0.

seen that the depth of the dip in the absorption is determined by γ_0, which is simply the lifetime of the dark state coherence $\left|-\right\rangle = \cos\theta\left|1\right\rangle - \sin\theta\left|2\right\rangle$.

B. Experimental Demonstration

A nice experimental demonstration of these phenomena was made by A.S. Zibrov et al. in a gas cell of ^{87}Rb [12]. The three level system was realized in the D1 transition by coupling the $F_g = 1$ and $F_g = 2$ hyperfine ground state manifolds of the $5S_{1/2}$ level to the single upper state hyperfine manifold $F_e = 2$ in the $5P_{1/2}$ level with right-hand circularly polarized light near 795

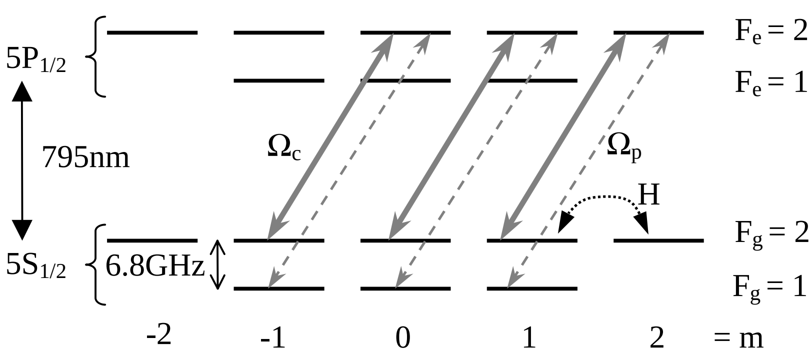

Fig. 3: The energy levels for the D1 transition in ^{87}Rb. Three Λ-systems are created with rhcp light. The thick and dashed arrows represent the strong coupling and weak probe fields, respectively. H is a weak magnetic field used to couple population out of the $F_g = 2$, $m_{Fg} = 2$ state. the repump field is not shown.

nm, as shown in Fig. 3. In this way three degenerate Λ systems are created. Note that linearly polarized light could not be used because the ($F_g = 2, m_{Fg} = 0$) to ($F_e = 2, m_{Fe} = 0$) transition is dipole forbidden, meaning that a strong coupling cannot be created on this transition, so there would be strong absorption on the ($F_g = 1, m_{Fg} = 0$) to ($F_e = 2, m_{Fe} = 0$) transition. Also, a weak magnetic field H was applied so that atoms would not become trapped in the ($F_g = 2, m_{Fg} = 2$) state. These details are mentioned to highlight some of the issues which were ignored in the theoretical model above.

Zibrov et al. measured the transmission of the probe beam with a simple photodetector, and the phase lag with a Mach-Zehnder interferometer. They also applied a weak incoherent (150 MHz linewidth) repump beam to redistribute population out of the lowest state. The atomic number density was relatively high, ~10^{12} cm^{-3}. This was done to accentuate the high transimissivity of a medium which would normally completely absorb the 5 μW probe. The 10 mW coupling field was resonant with the one photon transition ($\Delta = 0$), and the probe was swept through the Raman resonance, giving the data shown in Fig. 4, which agrees with the narrow dip in the absorption and the attendant strong dispersion predicted by Eq. 7. It should be

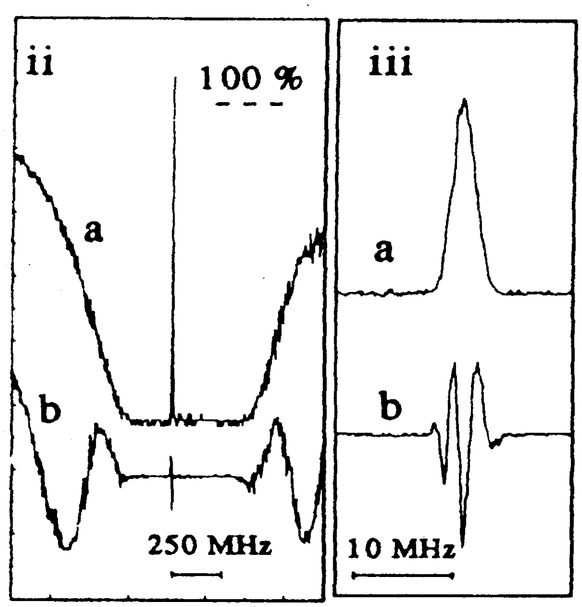

Fig. 4. Experimental demonstration of EIT by A.S. Zibrov et al. Trace **a** is the transmission of the probe field as a function of detuning. Trace **b** is the phase shift measured by interferometry. Box **iii** is a close-up of box **ii**.

noted that the transmission of the probe exceeds 100% on Raman resonance. This is due to the repump beam, and demonstrates the phenomenon of gain without inversion, the essence of LWI [1].

Another fascinating experiment which takes advantage of the first order susceptibilities in EIT was performed by L.V. Hau et al. [4]. Noting that the group velocity of a pulse is given by

$$v_g = \frac{c}{n(\omega_p) + \omega_p \frac{\partial n}{\partial \omega_p}}, \tag{8}$$

Hau et al. used the steep variation in the refractive index associated with EIT (as in Fig. 2(a)) to dramatically reduce the speed of a pulse in a gas of ultracold sodium atoms. Since the absorption was almost zero due to EIT they were able to cool the sample to 50 nK in a 4 Dee trap, creating an almost pure Bose-Einstein condensate (≥90% of atoms in 4 Dee trap ground state), and thus a very high atomic density. They show that a pulse travelling through the 229 μm long gas cloud was delayed by 7.05 μs from a pulse traversing free space, indicating that the light speed in the cloud was an amazing 32.5 m/s. Because these pulses are moving so slowly, they are interacting with the cloud for times approaching the dephasing time γ_0^{-1}; Hau et al. note that this sets the ultimate limit on the propagation rate of the pulses.

Another interesting aspect of this experiment is that these pulses are spatially compressed: the 2.5 μs pulse used by Hau is 750 m long in free space, but only 42 μm long in the cloud. S.E. Harris et al. [9] note that this creates very high optical energy densities in the cloud, and thus allows one to observe nonlinear phenomena at very low light intensities.

III. THE C-NUMBER LANGEVIN APPROACH

The semiclassical model in the adiabatic limit gives an excellent introduction, and illustrates many of the essential features associated with EIT. But it has been shown that in the adiabatic limit the model ignores some of the higher order effects of EIT. Specifically, a perturbative approach demonstrates that the applied fields can develop phase correlations due to the nonadiabatic response of the atomic system (the atoms have a finite ‘memory’). In this section a perturbative approach is used in the c-number Langevin formalism to describe these effects [5].

With the atomic operators σ_{jk} defined in the usual way for density matrix elements we write the Hamiltonian for a single atom in the rotating wave approximation as

$$\begin{aligned}\hat{H} &= \hat{H}_0 + \hat{I}^{(s)} \\ \hat{H}_0 &= \hbar\{\omega_1\hat{n}_1 + \omega_2\hat{n}_2 + \varepsilon_3\hat{\sigma}_{33} + \varepsilon_1\hat{\sigma}_{11} + \varepsilon_2\hat{\sigma}_{22}\} \ , \\ \hat{I}^{(s)} &= \hbar\left\{g_1\left(\hat{a}_1^+\hat{\sigma}_{13} + \hat{a}_1\hat{\sigma}_{31}\right) + g_2\left(\hat{a}_2^+\hat{\sigma}_{23} + \hat{a}_2\hat{\sigma}_{32}\right)\right\}\end{aligned} \tag{9}$$

where $\hat{n}_i = \hat{a}_i^+\hat{a}_i$ is the number operator for the i^{th} field, ε_j is the energy of the j^{th} level, and g_i is the coupling strength given by

$$g_i = \sqrt{\frac{\hbar\omega_i}{2\varepsilon_0 AL}}\frac{\wp_{3i}}{\hbar}, \tag{10}$$

in which A is the beam cross section, L is the interaction length. The superscript (s) denotes the Schrödinger picture. The Hamiltonian is recast in the interaction picture, giving

$$\hat{I}(t) = \hbar\left\{g_1\left(\hat{a}_1^+\hat{\sigma}_{13}e^{i\Delta t} + \hat{a}_1\hat{\sigma}_{31}e^{-i\Delta t}\right) + g_2\left(\hat{a}_2^+\hat{\sigma}_{23}e^{i(\Delta+\delta)t} + \hat{a}_2\hat{\sigma}_{32}e^{-i(\Delta+\delta)t}\right)\right\}. \tag{11}$$

The time evolution of the atomic operators follows from

$$\dot{\hat{\sigma}}_{jk} = \frac{1}{i\hbar}\left[\hat{I}(t), \hat{\sigma}_{jk}\right] + \hat{R}_{jk} + \hat{F}_{jk}(t), \tag{12}$$

where $F_{jk}(t)$ are quantum noise operators which are used to include the phase diffusion effects of coupling to the vacuum reservoir [5]. The fluctuation forces for different atoms are uncorrelated. The atomic relaxation rates are considered much slower than those of the reservoir as in the Markov approximation [1]. The noise operators have zero mean value, and are delta-function correlated in time

$$\left\langle F_{jk}(t)F_{j'k'}(t')\right\rangle = D_{jkj'k'}\delta(t-t') \ , \tag{13}$$

where $D_{jkj'k'}$ is the single atom diffusion coefficient. With this Hamiltonian we can write the equations of motion for the atomic operators as

$$\dot{\hat{\sigma}}_{33} = -(\gamma_1 + \gamma_2)\hat{\sigma}_{33} - ig_1\left(\hat{a}_1^+\hat{\tilde{\sigma}}_{13} - \text{H.a.}\right) - ig_2\left(\hat{a}_2^+\hat{\tilde{\sigma}}_{23} - \text{H.a.}\right) + \hat{F}_{33}(t) \ , \tag{14}$$

$$\dot{\hat{\sigma}}_{11} = \gamma_1\hat{\sigma}_{33} + ig_1\left(\hat{a}_1^+\hat{\tilde{\sigma}}_{13} - \text{H.a.}\right) + \hat{F}_{11}(t) \ , \tag{15}$$

$$\dot{\hat{\sigma}}_{22} = \gamma_2\hat{\sigma}_{33} + ig_2\left(\hat{a}_2^+\hat{\tilde{\sigma}}_{23} - \text{H.a.}\right) + \hat{F}_{22}(t) \ , \tag{16}$$

$$\dot{\hat{\tilde{\sigma}}}_{12} = (-i\delta - \gamma_0)\hat{\tilde{\sigma}}_{12} - ig_1\hat{a}_1\hat{\tilde{\sigma}}_{32} + ig_2\hat{a}_2^+\hat{\tilde{\sigma}}_{13} + \hat{F}_{12}(t) \ , \tag{17}$$

$$\dot{\hat{\tilde{\sigma}}}_{13} = \left(i\Delta - \frac{\Gamma}{2}\right)\hat{\tilde{\sigma}}_{13} - ig_1\hat{a}_1(\hat{\sigma}_{33} - \hat{\sigma}_{11}) + ig_2\hat{a}_2\hat{\tilde{\sigma}}_{12} + \hat{F}_{13}(t) \ , \tag{18}$$

$$\dot{\hat{\tilde{\sigma}}}_{23} = \left(i(\Delta + \delta) - \frac{\Gamma}{2}\right)\hat{\tilde{\sigma}}_{23} - ig_2\hat{a}_2(\hat{\sigma}_{33} - \hat{\sigma}_{22}) + ig_1\hat{a}_1\hat{\tilde{\sigma}}_{21} + \hat{F}_{23}(t) \ , \tag{19}$$

where a transformation has been made to frame rotating as in Eq. 1. For correspondence, it can be shown that these equations of motion reduce to the semiclassical equations (Eq. 4) with the substitutions $a_i = 1$, $g_i = -\Omega_i/2$.

In order to describe the propagation of the two fields through the sample we follow the approach of P.D. Drummond et al. [13] in which the quantized field operators $\hat{a}_{1,2}$ are replaced by space and time dependent complex amplitudes $\alpha_{1,2}(z,t)$ which obey the Maxwell equations in the slowly varying amplitude and phase approximation,

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\alpha_i(z,t) = ig_i N\sigma_{i3}(z,t) \ . \tag{20}$$

where N is the mean number of atoms in the sample, and $\sigma_{13}(z,t)$ and $\sigma_{23}(z,t)$ are continuous versions of the atomic operators, and can be derived by summing over the individual atoms and positions. Since these are sums of the individual atomic operators, they obey the same equations of motion as the individual atom operators, but the noise operators must be modified to include the new spatial dependence, giving the correlations

$$\left\langle F_{jk}(z,t)F_{j'k'}(z',t')\right\rangle = D_{jkj'k'}\frac{L}{N}\delta(z-z')\delta(t-t')\ . \tag{21}$$

After making these substitutions into the equations of motion, the equations of motion and the Maxwell equations may be solved by assuming that both the atomic and field variables have a time dependence which only causes small fluctuations about their steady state values: $x(z,t) = \bar{x}(z) + \delta x(z,t)$. For simplicity we take the fields to be on resonance for both the one and two photon transitions ($\Delta = \delta = 0$). To find the steady state behavior we drop the time derivatives and quantum noise fluctuations, which reduces the equations of motion to algebraic expressions. After a modicum of algebra, the resulting expressions for $\bar{\sigma}_{13}(z)$ and $\bar{\sigma}_{23}(z)$ are used in the Maxwell equations, giving

$$\begin{gathered}\frac{\partial}{\partial z}\bar{\alpha}_{1,2}(z) = -\frac{2g_1^2 g_2^2 N\gamma_0}{Dc}\gamma_{1,2}\bar{n}_{2,1}(z)\bar{\alpha}_{1,2}(z) \\ D \equiv 12\gamma_0 g_1^2 g_2^2 \bar{n}_1(z)\bar{n}_2(z) + \left(g_1^2\gamma_2\bar{n}_1(z) + g_2^2\gamma_1\bar{n}_2(z)\right)\left[\gamma_0\Gamma + 2\left(g_1^2\bar{n}_1(z) + g_2^2\bar{n}_2(z)\right)\right]\end{gathered} \tag{22}$$

where $\bar{n}_{1,2}(z) \equiv \left|\bar{\alpha}_{1,2}(z)\right|^2$. In Eq. 22 it can be seen that in steady state the propagation of one field depends only on the amplitude of the other field. Thus there is no coupling of the field phases in steady state. Furthermore it can be seen that the absorption rate

$$\kappa_{1,2}(z) = 4g_1^2 g_2^2 N\gamma_0\gamma_{1,2}\bar{n}_{1,2}(z)/Dc \tag{23}$$

approaches zero as γ_0 approaches zero, as was seen earlier (for $\delta = 0$). These are the same results derived in section II.

We next solve the linearized equations for the small fluctuations $\delta x(z,t)$. The algebra is simplified by restricting the analysis to Fourier frequencies smaller than the fast atomic decay rates [5]. This limit is taken by making an adiabatic elimination of the atomic variables which decay with the rate γ_1 or γ_2, namely, σ_{33}, σ_{13}, σ_{23}, and the sum of the lower level populations $\sigma_{11} + \sigma_{22} = 1 - \sigma_{33}$. It should be noted that this limit is not really a restriction; we are most interested in low frequency fluctuations since Fourier frequencies outside the atomic linewidth interact only weakly with the system. We then write the field variables as

$$\alpha_{1,2}(z,t) = \left|\alpha_{1,2}(z,t)\right| e^{i\phi_{1,2}(z,t)} \quad . \tag{24}$$

The differential equations for the remaining atomic variables can be transformed to a set of algebraic equations by making a Fourier transformation [5]. The results are substituted into the field equations, giving the following expression which governs the propagation of the phase difference between the fields $\delta\phi(z,\omega) = \delta\phi_1(z,\omega) - \delta\phi_2(z,\omega)$

$$\frac{\partial}{\partial z}\delta\phi(z,\omega) = -\frac{1}{2}\left[\kappa_\phi(z) - i\tilde{\kappa}_\phi(z)\right]\delta\phi(z,\omega) + F_\phi(z,\omega) \quad . \tag{25}$$

where the phase difference damping rate $\kappa_\phi(z)$ is

$$\kappa_\phi(z) = \frac{8g_1^2 g_2^2 N(\bar{n}_1(z) + \bar{n}_2(z))}{\Gamma^2 \Gamma_g c} \frac{1}{1 + \frac{\Gamma_g^2}{\omega^2}} \tag{26}$$

$$\Gamma_g = \gamma_0 + 2\frac{\left(g_1^2 \bar{n}_1 + g_2^2 \bar{n}_2\right)}{\Gamma} \quad . \tag{27}$$

The term at the far right of the expression for κ_ϕ can be recognized as a Lorentzian dip of width Γ_g, causing zero damping at $\omega = 0$. This is the EIT feature; note that if we make the semiclassical substitution $g_i = -\Omega_i/2$, and take one of the fields to be much stronger than the other, then Eq. 27 is identical to the semiclassical result for $\Delta\omega_{EIT}$. This expression shows that phase differences between the two fields which occur at a Fourier frequency outside the EIT width are strongly damped as the fields propagate through the gas cell. This can be understood by considering that these fast phase differences occur at a rate faster than the superposition state can follow adiabatically. Thus these frequency components are absorbed by the atomic system. It is this phenomenon which gives rise to the pulse matching of two pulses on resonance with a three level system [14].

The stabilization of the phase of two initially uncorrelated fields suggests using this system to generate a stable beatnote between two lasers. The stability is ultimately set by the width of the EIT feature, which may be reduced by reducing the field amplitudes. However, it can be seen from Eq. 22 and Eq. 23 that as the Rabi frequencies decrease, the absorption increases. The effectiveness of the system in reducing the phase difference fluctuations is determined by the ratio of the phase difference damping length $1/\kappa_\phi$ to the absorption length $1/\kappa_{1,2}$ [5].

IV. SUMMARY

A review of the optical properties of EIT has been given in the semiclassical picture. The dark state atomic coherence was derived from the Hamiltonian of a Λ-type atomic system interacting with two fields. The dark state atomic coherence was shown to inhibit absorption on Raman resonance, and to allow control of the ground state populations. An experiment was reviewed which demonstrated that the dark state can be used to increase the efficiency of nonlinear processes such as frequency conversion. The relevant first order susceptibilities were calculated and shown to exhibit strong dispersion in the absence of absorption. An experiment which clearly demonstrated these susceptibilities was reviewed, illuminating some of the details of EIT in real atomic systems. This experiment also demonstrated the role EIT plays in LWI. The application of the strong dispersion in the absence of absorption associated with EIT to ultraslow light propagation and novel regimes of nonlinear optics was introduced. The nonadiabatic effects have been calculated in the c-number Langevin formalism, and shown to give rise to phase correlation between the incident fields. Applications of this effects to the Raman clock were discussed.

Overall, EIT describes a novel set of optical properties, the essence which can be calculated quite easily. Researchers have found a wide variety of applications for this phenomenon, and new applications will undoubtedly be reported soon [15].

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