Optical soliton: classical *vs* quantum characteristics

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Optical solitons have potential for application in many fields, including the telecommunications industry. The classical characteristics of solitons are reviewed. Despite their obvious macroscopic size, solitons exhibit quantum mechanical features. In particular, soliton squeezing can reduce the noise to below the shot noise levels.

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I. INTRODUCTION

The first recorded sighting of a soliton was made by John Scott Russell in 1844. He observed a heap of water in a canal which propagated for a distance of one or two miles without losing its shape. Others worked on the problems in subsequent years and even confirmed Russell's observations. The optical pulse which is called a soliton is not a solitary wave in the sense of Russell's water wave since there exists many EM periods under the pulse envelope. It is the envelope itself which exhibits solitonic characteristics. Henceforth, it is understood that we refer to the envelope when we talk of optical soliton. Furthermore, the study is restricted to temporal solitons though there is a great deal of interest in spatial solitons as well. Many general sources exist for the reader interested in gaining greater depth of understanding of this subject [1, 2, 3, 4].

An optical pulse travelling in any medium other than vacuum will tend to broaden since its frequency components generally have different group velocities in the medium. This effect is known as chromatic dispersion or group velocity dispersion (GVD). In the positive GVD regime, the longer wavelengths (red) travel faster than the shorter wavelengths (blue). A stable pulse can exist if the different pulse broadening mechanisms work against each other such that the net dispersion is nulled. For example, pulses of sufficient power can take advantage of the nonlinear Kerr effect to offset the chromatic dispersion and thus are able to maintain their shapes. One such nonlinear effect is self-phase modulation (SPM). The pulse has an intensity gradient along the direction of propagation, resulting in a phase gradient along that axis which is proportional to $n_2 I(t)$. In effect, the pulse's spectral content changes as it travels. When n_2 is positive, the leading edge of the pulse gets shifted to longer wavelengths and the trailing edge is shifted to shorter wavelengths. Both GVD and SPM by themselves cause pulse broadening, but they can cancel out if the pulse parameters are chosen properly, e.g. negative GVD and positive n_2 with the net phase changes equal to zero.

The characteristic lengths L_D and L_{NL} are the length scales over which chromatic dispersion and the nonlinear interactions have significant effects on the pulse. The balancing act between L_D and L_{NL} sometimes result in solitonic propagation and determines many of the characteristics for the soliton. The wave can withstand minor perturbations without losing its form, continually adjusting itself so as to remain a solitary wave. One consequence of such modifications is that the central frequency shifts resulting in a change of the pulse's velocity, thus affecting the time of arrival. A train of pulses initially equally separated in time can arrive at its destination at different time separations when the pulses' amplitudes vary or if their central frequencies differ.

The classical soliton is treated as a robust phenomenon, able to withstand perturbations without changing its form by much. However, that understanding is based on a simple picture in which the soliton is allowed to travel without loss. A common way to transmit light is the fiber optic waveguide made of fused silica. Though losses are kept very low with improvements in manufacturing techniques, they exist. Any loss implies that SPM will eventually not be able to compensate for GVD broadening. Reamplification of the pulse train is the obvious solution, but that introduces yet other issues such as the Gordon-Haus (G-H) effect. The amplifier makes no distinction between signal and noise and will reamplify both. In addition, the gain medium will give off spontaneous emission as well as stimulated emission. This additional noise further degrades signal and also results in G-H timing jitter. The jitter could accumulate to the extent that it destroys the signal that one wishes to send, espescially when the distances approach hundreds and thousands of kilometers. A self-consistent, robust quantum mechanical description of solitons has not been realized. However, the various models proposed show promise, or at least, give insight to the quantum nature of solitons.

The statistics of the optical soliton follows that of a coherent source. That is, if one were to measure the number of photons $\mathcal{N}$ that are in the pulse, there will be fluctuations in that measurement which follow Poissonian statistics. This fluctuation or noise is known as the shot noise and is proportional to the mean number of photons $\bar{\mathcal{N}}$ per pulse for coherent sources. It was discovered that quantum effects do show up in soliton propagation despite the macroscopic nature of solitons. Solitons' peak power have to be large in order to take advantage of the nonlinear effects in the material hence $\bar{\mathcal{N}}$ has to be very large. Typical values are of the order of 10^8 or greater. The noise of the measurements is an undesirable effect for most applications so much work has been done to reduce it. The quantum mechanical concept of squeezing is used to improve the signal-to-noise ratio for various measurements. Previous reviews of this field of study include [5, 6, 7]. They are highly recommended for a more thorough introduction to quantum solitons.

II. THEORETICAL SUMMARY

A. Classical soliton

The classical optical soliton starts out as a solution to the simplified wave propagation equation which takes the form of the nonlinear Schrödinger equation (NLSE)

$$i\frac{\partial u}{\partial \xi} = \mathrm{sgn}(\beta_2)\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} - |u|^2 u, \tag{1}$$

where u is the field amplitude normalized by the pulse peak power P_0, ξ is the position variable normalized to the dispersion length L_D, τ is the time variable normalized to the pulse width T_0, β_2 is the group velocity dispersion (GVD) of the material, and $\mathrm{sgn}(\beta_2) = \pm 1$ with the positive value for the normal dispersion regime. The parameter N hidden in the function u summarizes the interplay between the dispersion and the nonlinear action. It is called the soliton number and is given by the following relation

$$N^2 = \frac{L_D}{L_{NL}} = \frac{T_0^2/|\beta_2|}{1/\gamma P_0} = \frac{\gamma P_0 T_0^2}{|\beta_2|}. \tag{2}$$

The nonlinearity coefficient γ takes into account the nonlinear index of refraction n_2, the light frequency ω_0 and the effective pulse cross-sectional area A_{eff} and is defined as

$$\gamma = \frac{n_2 \omega_0}{c A_{eff}}. \tag{3}$$

The exact solution for the $N = 1$ soliton in the anomalous GVD regime is [8, 1]

$$u(\xi, \tau) = 2\eta_1 \text{sech}(2\eta_1 \tau) \exp(2i\eta_1^2 \xi). \tag{4}$$

One can normalize the soliton such that $u(0, 0) = 1$ to give the canonical form

$$u(\xi, \tau) = \text{sech}(\tau) \exp(i\frac{\xi}{2}). \tag{5}$$

However, in order to make use of real soliton systems, a less simplistic model is needed. Other effects come into play such as material absorption (which can be compensated for by introducing a gain medium along the propagation path), Raman fluctuations, guided acoustic wave Brillouin scattering (GAWBS), and so forth [9]. These are observed experimentally before theory can predict their behaviors. The empirical observation of these phenomena implies the need to model and understand how they interact with light, and more importantly, to control these effects if not make use of them in some useful application. Several schemes exist to manipulate soliton including four-wave mixing and parametric amplification as well as variants of these approaches.

B. Amplification

Parametric amplification is a mean to reveal the quantum nature of solitons. Deutsch and Abram [10] proposed the scheme of phase sensitive amplification (PSA) which would reduce the phase noise introduced to soliton propagation. Phase insensitive amplification (PIA) increases the signal but adds excess noise as well. The added noise is random in nature and affects both phase quadratures equally. Hence the noise increase in PIA is both through the gain factor and the (amplified) spontaneous emission of the gain medium. Theory predicts that it grows, in the asymptotic limit, as the cubic power of the distance of travel. PSA does not add this excess noise to the system and can squeeze one of the phase quadratures if the gain parameters are well chosen. The jitter grows linearly in the asymptotic limit as well which greatly improves the signal over that of the PIA case. The asymptotic limit is taken as the distance which is much greater than the inverse of the fiber loss rate.

PSA is a viable method for managing quantum noise. At least one such amplifier is claimed to exist [11]. Researchers associated with the device recently proposed a method to store bit stream using PSA [12]. For long term storage, one has to keep the signal noise-free continually or else the stored data cannot be retrieved without error.

The origin of the G-H effect is the increased frequency fluctuation introduced by the amplifier itself. Recall that amplitude fluctuations give rise to frequency fluctuation from the nonlinear phase modulation processes. The soliton continually reshapes itself to maintain its form as it travels and the added frequency noise means that the pulse's central frequency is also more noisy. If the wave guide is optical fiber, for example, GVD will force these pulses to travel at different speeds, hence the pulse timing jitter. Various techniques are known to help keep this noise manageable such as bandpass filters. These impose a particular range of frequencies on the pulse train, forcing excess loss for the frequency components outside the range. There is also the sliding/guiding filter which gently eases the soliton frequency from one value to another. The soliton component of the pulse will be able to readjust itself to keep up with the changing frequency while the nonsoliton part will be left behind and eventually removed from the system. At the worst case, it is allowed to propagate without overlapping with the solitons.

C. Four-wave mixing and QND

Four-wave mixing has been used to introduce squeezing of the soliton. Levenson and Shelby [13] showed how the coupling between different modes of the radiation field can affect the noise associated with one of the modes. In essence, they proposed a method in which the Kerr nonlinearity in optical fiber can be used to reduce phase noise as well as be incorporated into quantum nondemolition (QND) measurements. The cross-phase modulation (XPM) between the waves can introduce phase shifts given by

$$\delta\phi_x(l) = \delta\phi_x(0) + \frac{4\pi l}{\lambda} n_2[\langle E_x\rangle \delta E_x + 2\langle E_y\rangle \delta E_y], \tag{6}$$

$$\delta\phi_y(l) = \delta\phi_y(0) + \frac{4\pi l}{\lambda} n_2[2\langle E_x\rangle \delta E_x + \langle E_y\rangle \delta E_y]. \tag{7}$$

Notice that the introduced phases are expressed as phase fluctuations, with the subscripts indicating the two different waves. As a realization of this effect, consider two copropagating solitons of different velocities which are allowed to collide [14]. The phase shift of one pulse, say the probe, reflects the amplitude, effectively the photon number, of the other pulse. By monitoring the phase fluctuations of the probe, it then is possible to deduce the number fluctuations, hence the mean photon number, of the signal pulse. The collision does not decrease the photon number of the signal which would be the case if one were to use a beamsplitter or outcoupler in the system. Nor does it introduce additional photon number noise to the signal, hence the term "nondemolition" in QND.

D. Quantum model

Drummond and Carter [15] presented a quantum treatment of propagation which predicts squeezing of the continuous wave as well as the soliton. Haus et al. [14, 16] later proposed a quantum theory for optical soliton. The authors treat the field quantization in terms of the soliton modes and the continuum. They explain soliton squeezing by using a linear approximation of the field operators such that the field is comprised of a classical soliton solution perturbed by a quantum mechanical field. It is the quantum perturbing field which give rise to the fluctuation operators (photon number $\Delta\hat{n}$, phase $\Delta\hat{\theta}$, momentum $\Delta\hat{p}$, and position $\Delta\hat{x}$). Furthermore, the product of the photon number fluctuation and phase fluctuation is not equal to the usual Heisenberg uncertainty quantity. That is, the theory predicts that the vacuum fluctuation would not give rise to a minimum uncertainty state. The calculations show that the fluctuations evolve as follows

$$\Delta\hat{n}(t) = \Delta\hat{n}_0, \tag{8}$$

$$\Delta\hat{\theta}(t) = \Delta\hat{\theta}_0 + \frac{n_0|c|^2}{2}\Delta\hat{n}_0 t, \tag{9}$$

$$\Delta\hat{p}(t) = \Delta\hat{p}_0, \tag{10}$$

$$\Delta\hat{x}(t) = \Delta\hat{x}_0 + 2\Delta\hat{p}_0 t. \tag{11}$$

Two of these fluctuations are constant in time and the other two have a linear time dependence the slope of which is determined by the fluctuations of their conjugate variable. Already, squeezing is predicted in the evolution of the fluctuation operators. They also outlined the technique to measure the expectation values of these four fluctuation operators using a homodyne detection scheme. A discussion on homodyne and heterodyne detection schemes is given by Yuen and Chan [17]. The paper essentially outlines the mathematical aspect of the balanced detector and presents some challenging fundamental ideas to reflect upon.

There are many features of soliton propagation and interaction that merit study for themselves. Werner and colleagues [18, 19, 20] explore some of the more esoteric avenues that come from soliton studies. The push amongst some is for a more unified quantum soliton theory. More resources ought to be spent on detailed experimental and theoretical studies since they might point the way to self-consistency instead of searching for immediately useful devices.

III. EXPERIMENTS

Many groups have looked at the quantum mechanical aspects of the soliton. Often it is difficult to see quantum nature since the soliton *is* a large, macroscopic entity and most experimental studies of the temporal soliton involve optical fiber in some way. The fiber itself helps to create the soliton in the first place, but its other classical interactions with the light field further bury the quantum signatures in more noise. Despite the difficulties, only a little over a decade passed between the observation of optical solitons and the explicit verification of their quantum nature.

A. Quadrature amplitude squeezing

Rosenbluh and Shelby [21] are the first to publish experimental results which showed quadrature-amplitude squeezing due to optical solitons. They reported squeezing which took the photocurrent noise power 1.7 dB below the shot noise limit at T = 77 K and 1.1 dB at room temperature. Their

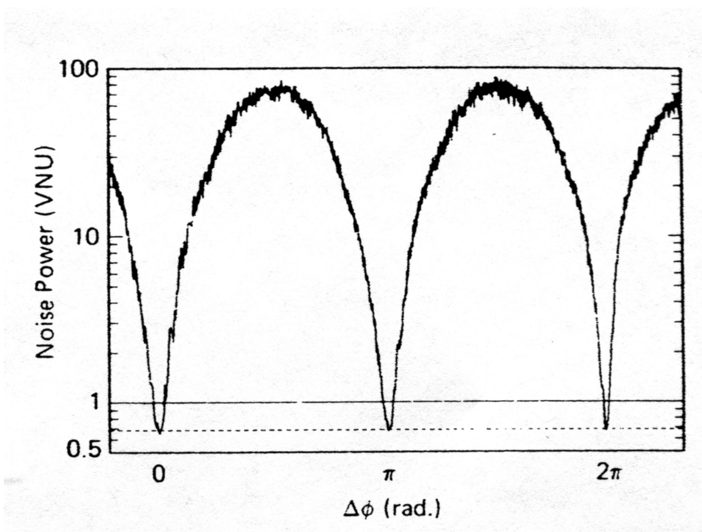

Figure 1: Photocurrent difference noise power, normalized to the vacuum level noise power in the same bandwidth is shown as a function of the relative LO phase, $\Delta\phi$. The noise power normalized thus is in vacuum noise units, or VNU. The horizontal solid line is the vacuum noise level or classical shot-noise limit, while the dotted line is the minimum noise of the squeezed soliton of 0.68 VNU. These data were recorded for a pulse energy traveling in each direction in the fiber of 0.21 nJ. The resolution bandwidth of the spectrum analyzer was 30kHz, the video bandwidth was 30 Hz, and the sweep time was 8 s [21].

result[1] at 15 MHz is shown in Fig. 1. There appeared to be an external limit to the extent of the observed squeezing which was attributed to GAWBS and to Raman effect. Kärtner et al. [22] gave a model which supports the Raman hypothesis. Their model predicted a limitation to squeezing in Raman amplifier systems. This theory, if valid, presents a dilemma since many in the telecom-

[1] Noise measurements are often quoted for a particular frequency and bandwidth.

munications field would like to compensate for fiber losses utilizing Raman amplification. Such an amplifier has advantages over discrete amplifiers such as Erbium-doped fibers.

The experimental methodology of Ref. [21] was proposed by Shirasaki and Haus [23] in which a Mach-Zehnder type interferometer is used in a balanced detection scheme to separate out the classical and the quantum effects of pulse propagation. The balanced detection allowed the two signals to be subtracted from each other, leaving only the results due to quantum effects. A particularly simple implementation of their scheme is similar to the nonlinear optical loop mirror used in some passively modelocked fiber lasers. The fiber 3 dB couplers act as beamsplitters (BS) to split up the beams and to recombine them as well. The schematic from their paper is shown in Fig. 2.

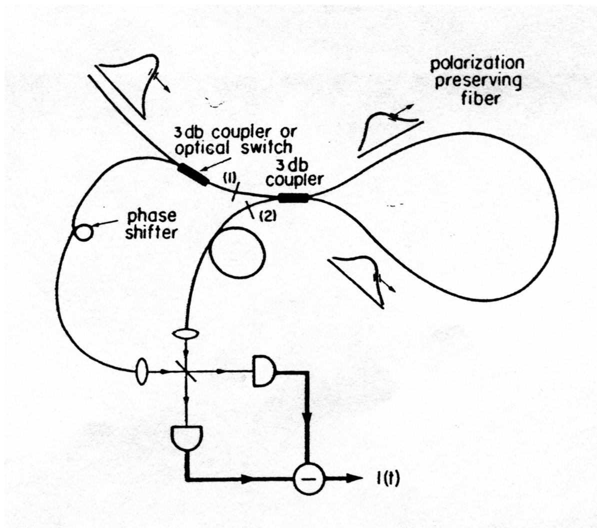

Figure 2: Schematic of the experimental setup [23].

The soliton passes through the first coupler with very little light coupled into the arm with the phase shifter. Its amplitude is equally divided by the second 3 dB coupler and allowed to traverse the loop and be recombined at the same coupler on its way out. In the linear regime, the two pulses experience the same phase shifts, and will interfere destructively at output port 2 and constructively at output port 1, which is also the input port. The first coupler picks off some of the reflected light and sends it to the balanced detector as the local oscillator LO signal. The zero-point fluctuations is considered to originate from port 2 and enters the loop in counterpropagating directions as well and reemerge also along port 2. The nonlinear action comes in when the strong soliton pump interferes with the weak vacuum fluctuations while both are in the loop and introduces squeezing on the vacuum fluctuations which now come out along port 2. This squeezed signal is recombined on a BS with the LO, appropriately delayed and phase adjusted, in the balanced detector. The difference of the two electric signals give the squeezed light signal alone.

B. Photon number squeezing and other advances

QND was the second quantum mechanical effect of solitons to be observed [24] in 1992. The same group then demonstrated soliton photon number squeezing [25]. They launched $N > 1$ solitons into optical fiber with anomalous GVD and allowed the pulses to broaden spectrally. After a length of several soliton periods, defined as $z_0 = \frac{\pi}{2} L_D$, the pulses exit the fiber and the wings of the spectral profile are removed (i.e. with a bandpass filter). The reported photon number squeezing is as much as 3.7 dB. Their quantum model showed that the number fluctuations are greater in the wings than in the central spectral region and support the clipping of the outter frequencies for optimal squeezing.

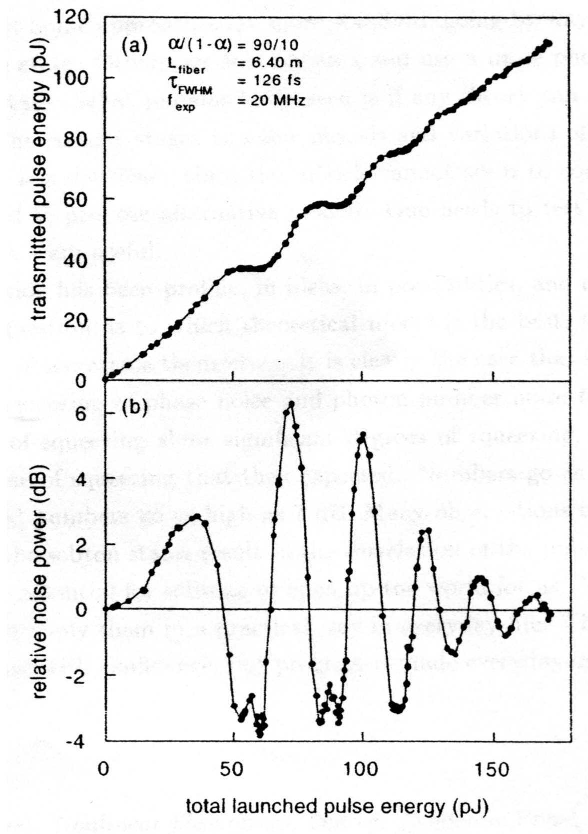

Figure 3: Nonlinear energy-transfer function and squeezing from a 90:10 asymmetric Sagnac loop, plotted versus the launched pulse energy. (a) The transmitted output pulse energy shows an optical limiting effect at input energies of 53 pJ and 83 pJ. (b)Photocurent noise power relative to shot noise (0 dB). The quantum fluctuations are reduced below shot noise at input energies where optical limiting occurs [26].

Photon number squeezing was again demonstrated [26] using the nonlinear fiber optic Sagnac interferometer to directly measure the photocurrent noise as compared to the shot noise. The interferometer setup was similar to that of Fig. 2 with the exception that the BS has a 10/90 splitting ratio. In addition, the local oscillator signal for homodyne detection as represented by the fiber length containing the phase shifter is not needed. Pulse energy fluctuations are observed as a function of the initial pulse energy. The best squeezing achieved noise reduction of 3.9 $\pm$ 0.2 dB below the shot noise limit shown in Fig. 3. The particularly exciting aspect of the setup is the relaxation on the loop BS constraints. A perfect 50/50 BS is not required for the first portion even though the detection BS still ought to be 50/50 for a balanced detector. Such an advance holds promise for more applications in the near future.

Recently, Werner [27] proposed that greater photon number squeezing is possible if one were to use longer fibers whose lengths are on the order of 100 z_0 and to cool the fiber to liquid He temperature. High-pass filtering of the signal results in the photon number squeezing of nearly

7 dB. This edge filter is a departure from previous experimental observations since most published works have utilized bandpass filters to optimize the number squeezing. The claims on the possible limits of number squeezing also runs counter to previously accepted models for soliton propagation. However, it is possible that the previous models do not work in the long propagation distance and high power regime that is suggested here. The paper is based on numerical calculations.

Shelby et al. [28] looked at several aspects of soliton squeezing, including the potential limitations in fiber. Their estimates of the GAWBS was that it extended over 20 GHz of bandwidth. This estimate is from their experiences with continuous wave squeezing. GAWBS adds excess noise due to the thermally induced vibrations in the waveguide (fiber). This noise show up as sidebands on the phase-noise measurements. Poustie [29] later revised the bandwidth estimate to ≈ 1.5 GHz. Bergman et al. [30] proposed a clever method to eliminate GAWBS induced phase noise. A portion of the soliton is picked off and delayed before sending the pair of pulses into the experimental setup of [23]. The ingenuity lay in shifting the phases of the two LO pulses by π with respect to each other before they are recombined with the squeezed signal. The time delay is chosen such that the pulses are close enough so that they experience the same acoustic vibrations, yet far enough apart so that the π phase shifter can distinguish the two pulses. The recombining BS before the balanced detector is replaced with a Mach-Zehnder interferometer (MZI) with one of the mirrors mounted on a PZT to vary the delay between the two arms of the MZI. Then the recombining BS is used in front of the balanced detector to measure the squeezing. They predicted a squeezing of 3.4 dB and actually measured around 3 dB of squeezing. The estimated reduction of the GAWBS induced noise is 8 dB. Imperfect cancellation of GAWBS noise is thought to be due to the technology only and not to physical limits. Townsend and Poustie [31] did a systematic study of the GAWBS suppression as a function of delay and depolarization effects. The delayed pulse technique has been reused by many groups, confirming its value as a tool in the field.

IV. CONCLUDING REMARKS

Solitons have very nice characteristics which could have many applications. However, they can present problems in completely unexpected ways. The nonlinear and the quantum aspects of these pulses require attention to details. Ultimately, it is possible that the idea of using solitons in long distance communication will be completely abandoned. In the meantime, many have contributed to the theoretical and experimental works.

Some theoretical approaches to quantization of the soliton state are standard, going by way of the P-representation to describe the it. Others are less rigorous and use a more phenomenological language to describe the same state. Predictions seem to be possible with some models for particular cases but not others. What remains to be seen is if any one theory can be pushed to include all regimes of interest. The current status is a few models and variations thereof. It is understood that there is a deficiency since the models cannot seem to converge.

The experimental side has been prolific, in ideas, in possibilities, and definitely in data. While there could be some question as to which theoretical model is the best, there doesn't seem to be any dispute about the observations themselves. It is clearly the case that solitons exhibit quantum characteristics, from squeezing of phase noise and photon number noise to various applications of QND. Measurements of squeezing show significant degrees of squeezing, but few groups felt like they reached the degree of squeezing that they expected. Numbers go as high as 3.9 dB for some experiments. Projected numbers go as high as 7 dB. Many observations of QND have been made.

The entanglement of the soliton states result in the correlation of the pulses. The technological limits have not been reached.

Solitons can open up another avenue for researchers. Much is yet to be learned from them and how to apply them in a practical way in everyday life. The process of learning about them bring forth many useful concepts that are applicable elsewhere. That learning alone is worth the study even if soliton should turn out to be impractical for direct usage.

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