A different approach to quantum decoherence

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(February 7, 2000)

Abstract

An overview of the still open problem of quantum decoherence is presented, with particular emphasis on the new approach developed by Bonifacio on intrinsic decoherence. A comparison with the popular model of environment-induced decoherence is made. Possible common characteristic are presented and analyzed.

PACS numbers: 03.65.Bz, 32.80,-t

I. INTRODUCTION

The central unsolved problem of quantum mechanics is the explanation of the transition from the laws which govern the micro-world to the ones of the macroscopic reality and its interpretation in the quantum theory of measurement [1]. These concerns appear as fundamental issues due to their philosophical implications regarding our view of the world.

The theory of quantum mechanics has been able up to now to explain and predict exceedingly well all the experimental evidences. Yet despite this spectacular success, there is still no consensus about its interpretation. The main problem centers around the inability to provide a natural framework of our observation of a "classical" world. The difficulties arise due to the fact that in the micro-world the principle of superposition, according to the Schrödinger equation, is valid while at the macroscopic scale this principle does not seem to exist.

In general the physical principles of classical mechanics cannot be applied to a quantum object. Nevertheless it is the natural tendency to look at quantum mechanics results via classical mechanics. Quantum concepts are in fact interpreted as a limiting case of the classical ones, similar to the limit of small velocities in relativity. From the deterministic linear Schrödinger equation, using Ehrenfest theorem, when a classical counterpart exists, the Newtonian mechanics can be recovered and the physical problem can be "simply" interpreted. The superposition principle, a consequence of the linearity of the Schrödinger equation, creates a certain uneasiness due to the conflict with everyday reality in which this principle seems to be violated. This absence of a classical counterpart has been typified by the famous example of the "Schrödinger cat", steered into a superposition of dead and alive.

Historically there have been essentially two explanations of how a single outcome emerges from the many possibilities: the Bohr's interpretation and the Everett's interpretation. For the first one quantum theory is not universal and the border line between the quantum and the classical is given by the measurement process in the sense that a classical apparatus is always necessary to make a measurement. The second one, the so-called many world's interpretation, states that the whole universe is described by quantum theory and each time a suitable interaction (measurement-like process) takes place between any two quantum systems the wave function of the universe splits in two branches. The world that we observe and in which we live is only one of them.

Both the interpretations rely on the Von Neumann postulate, stating that everytime a measurement is performed the wavefunction collapse to the eigenfunction of the measured observable. Furthermore, even if the two theories seem to be far apart, they have something in common. The many-world interpretation even if it avoids marking the limit between the quantum world and the classical one, in reality pushes it at the extreme border between the physical world and the consciousness.

Recent theoretical advances have stressed that the assumption of a closed system and therefore the applicability of the Schrödinger equation cannot be anymore justified [1]. Any object interacts with its environment and in order to determine the evolution, it has to be regarded as an open quantum system. The formalism of quantum theory uses the concept of a reduced density matrix for describing parts of a larger system. Technically, given a density operator which describes a small quantum system interacting with an external environment, all the associated external degrees of freedom are traced over. The system is consequently observed irrespective of the state of the environment and a reduced density operator is obtained. The superpositions of macroscopically different properties can be shown to disappear from these reduced density matrices on an extremely short time scale. This process is called "decoherence" [2].

It is worth furthermore to mention the *decoherent histories* theory developed recently by Gell-Mann and Hartle [1]. Starting from the validity of Everett's many-world interpretation, this theory looks for the requirements that must be put on a set of all possible alternatives, represented for example by a set of commuting projection operators, in order to obtain the "classicality" of familiar experience (absence of superposition of macroscopic states). The stringent requirements to be found, that give rise to decoherence, represent a coarse-graining of all the possible alternatives. Similar to the environment-induced decoherence analysis, that will be discussed in the next session, this theory tries to justify the Von Neumann state reduction postulate but from a much broader perspective. The type of abstraction is in a certain sense similar to approaching the study of classical mechanics through "global" variational principles compared to the local Hamiltonian ones.

The purpose of this paper is to discuss the approach to the problem of decoherence given by the quite popular theory originated from the work of Zurek [3] compared to the one of Bonifacio [4] appeared in a recent publication. According to the former one the unavoidable coupling to the environment is responsible for destroying the phase relations inside the quantum system. This process is responsible for the existence of an arrow of time: the information of the small system leaks out into the environment and is lost irreversibly.

Bonifacio's model is based on a mechanism of intrinsic decoherence in which the role of the environment is removed. A fundamental discretization of time is responsible for the irreversibility. This discretization is applied to the formalism of density matrices and in a consistent way it is shown that, without making any further assumption, the system, while evolving, decoheres.

The organization of this paper is as follows: in the next section (Sec. II) the general ideas and the approximations used to obtain decoherence in an open quantum system will be described. In Sec. III Bonifacio's model is explained. Section IV is devoted to the analysis of some experimental evidences which can confirm the theoretical derivation of Sec. III and in the last section, summarizing the results, a discussion will be opened.

II. ENVIRONMENT INDUCED DECOHERENCE

Any physical system realistically cannot be completely isolated from its environment. This is the starting idea that forces many scientists to believe that decoherence on a quantum system is due to the coupling to the external world. Zurek (1981) discussed the so-called environment-induced superselection rules. He was able to explain into what mixture the wave function of a system collapses. He stated that the form of the interaction Hamiltonian between an apparatus and its environment is sufficient to determine which observable of the measured quantum system can be considered "recorded" by the apparatus. The basis that contains this record (the pointer basis of the apparatus) consists of the eigenvectors of the operator which commutes with the apparatus-environment interaction Hamiltonian. The apparatus cannot be observed in a superposition of the pointer basis states because its state vector is being continually collapsed. The environment is "monitoring" the apparatus and this results in the localization of its wave function, that is to say decoherence.

In order to understand this picture, one has to agree with the facts that (a) the apparatus interacts with its environment via some specific interaction Hamiltonian and (b) the observer consults only the pointer of the apparatus and not the state of the environment. In this way the apparatus-environment interaction can then be regarded as an additional measurement establishing nonseparable correlations between the apparatus and the environment. As a result, information about the environment obliterates information about the just premeasured quantum system. However when the interaction Hamiltonian commutes with an observable of the apparatus, then this particular observable will not be perturbed. Only this particular pointer basis will contain the information about the quantum system itself. In other words if $|A_s\rangle$ and $|s\rangle$ are the pointer basis for the apparatus and the quantum system vector and $|A_0\rangle$ and $|\psi\rangle$ are the initial state of the apparatus and the system we can write

$$|A_0\rangle \otimes |\psi\rangle = \left\{\sum_s a_s |A_s\rangle\right\} \otimes \left\{\sum_s c_s |s\rangle\right\} \longmapsto \sum_s c_s |A_s\rangle \otimes |s\rangle. \tag{1}$$

This type of evolution is based on the following assumptions: (1) the interaction between the apparatus and the system acts only for a very short time and during that time the interaction of the apparatus with the environment is negligible, while afterwards this latter interaction becomes predominant and the system-apparatus Hamiltonian can be neglected, (2) all the vectors of the pointer basis correspond to the same degenerate energy eigenstates, (3) the environment and the "rest of the world" are considered not to interact as long as their interaction does not alter the interaction Hamiltonian between the apparatus and the environment, (4) all the interactions are pairwise and are measurement-like processes.

Assumption (1) seem clear and acceptable, (2) is the physical requirement that the measurement should not lead to the exchange of energy between the system and it is also a sufficient condition to be a nondemolition measurement for the observable diagonal in the $|A_s\rangle$ basis. Assumption (3) even if it seems customary based on physical argument states essentially that we are dealing somehow with a "closed" system once the degrees of freedom which do not influence the system-apparatus-environment are not taken into account. This requirement is probably necessary in order to single out a specific interaction Hamiltonian, which in turn means a specific pointer basis of the apparatus. It seems peculiar at least that in order to explain decoherent effects, the existence of an absolutely closed realistic system is denied but then this condition is restored later on. Assumption (4) is the important one on which the whole theory is based; it strongly states that there exist only a specific type of interaction between the apparatus and the environment.

In any case despite its popularity, this theory has not changed yet one of the postulates of quantum mechanics i.e. the collapse of the wave function once a measurement has been performed on the system, which, in the language of density matrices, implies that the off diagonal elements are abruptly put to zero. If the above assumptions are accepted, these so-called superselection rules can explain into what mixture the wave function collapse, but still do not explain why.

Along these lines it is interesting to mention the new proposed experiments of Haroche's group [5] in which the decoherence mechanism is viewed as a succession of uncontrolled and unread "measurement" of the system by its environment. The interesting claim is that, making a set-up in which the environment is made of a single quantum oscillator, it will be possible to show that decoherence due to the system-environment coupling, in such a controlled situation, becomes reversible. The irreversibility of physical reality is then obtained once the number of quantum oscillator becomes exceedingly large.

III. THE MODEL OF INTRINSIC DECOHERENCE

The starting idea of Bonifacio's model is to generalize the evolution equation for the density matrix of a quantum system. A generic system following unitary evolution is generally described by the Liouville-Von Neumann equation whose formal solution is

$$\rho(t) = e^{-iLt}\rho(0) \tag{2}$$

where ρ is the density operator, L the Liouvillian operator $L\rho \equiv \frac{1}{\hbar}[H,\rho]$ and H is the Hamiltonian. Bonifacio defines a generalized density operator $\bar{\rho}$ as

$$\bar{\rho}(t) = \int_0^\infty dt' P(t,t')\rho(t') \qquad t,t' \geq 0. \tag{3}$$

where $P(t,t')$ is a function to be determined. Requiring that the operator $\bar{\rho}(t)$ is a density operator and dropping the unitary evolution for the less restrictive assumption of semigroup property, it is possible to obtain for the evolution of the new density operator

$$\bar{\rho}(t) = \frac{1}{(1 + iL\tau_1)^{t/\tau_2}}\rho(0), \tag{4}$$

together with

$$P(t,t') = \frac{1}{\tau_1}\frac{e^{-t'/\tau_1}}{\Gamma(t/\tau_2)}(\frac{t'}{\tau_1})^{(t/\tau_2)-1} \qquad t,t' \geq 0. \tag{5}$$

Equations (4) and (5) tell us that time evolution occurs as a series of random events in which τ_2 is the average time step between two events and τ_1 is the time duration of each event. Taking $\tau_1 = \tau_2 \to 0$, $P(t,t') \to \delta(t - t')$ and equation (4) reduces to equation (2).

If the Hamiltonian of the system is independent of time, it is possible to derive explicitly Eq. (4) obtaining

$$\dot{\bar{\rho}}(t) = -\frac{1}{\tau_2} ln(1 + iL\tau_1)\bar{\rho}(t). \tag{6}$$

When τ_1 and τ_2 are finite, the second order expansion of Eq. (6) gives

$$\dot{\bar{\rho}}(t) = -i\frac{\tau_1}{\tau_2}L\bar{\rho}(t) - \frac{\tau_1^2}{2\tau_2}L^2\bar{\rho}(t). \tag{7}$$

where $L^2 = [H,[H,\rho]]$. Eq. (7) with $\tau_1 = \tau_2$ is the phase-destroying Master Equation (ME), whose double commutator has appeared in many models for coherence decay, deduced using particular reservoir interaction models or specific statistical assumptions [3,6].

From Eq. (4) it can be easily seen that $\bar{\rho}(t)$ obeys the following finite-difference equation:

$$\frac{\bar{\rho}(t) - \bar{\rho}(t - \tau_2)}{\tau_2} = -i\frac{\tau_1}{\tau_2}L\bar{\rho}(t) = -\frac{i}{\hbar}\left[\bar{H}, \bar{\rho}(t)\right] \tag{8}$$

where $\bar{H} = H\tau_1/\tau_2$. Eq. (8) again with $\tau_1 = \tau_2 \to 0$ gives the Liouville equation, while for $\tau_1 = \tau_2 = \tau$ it reduces to an equation proposed by the same author, almost twenty years ago, to describe irreversible state reduction to the diagonal form [7]. In ref. [8] it has been shown that for $\tau_1 = \tau_2$ Eq. (4) is equivalent to a semigroup ME of the Lindblad form.

Eq. (8) has an important characteristic: if $\bar{\rho}(t)$ is a solution, then any $f(t)\bar{\rho}(t)$ is also a solution provided that $f(t + \tau_2) = f(t)$. This means that $\bar{\rho}(t)$ is uniquely determined only within the time τ_2, that the author calls "cronon". Eq. (8) can be thought as if it provides the time evolution of $\bar{\rho}(t)$ for time intervals $t = k\tau_2$. $P(t,t')$ rewritten in terms of $t = k\tau_2$ is

$$P(k,t') = \frac{1}{\tau_1} e^{-t'/\tau_1} (\frac{t'/\tau_1{}^{(k)-1}}{(k-1)!}) \qquad t, t' \geq 0, \tag{9}$$

and can be interpreted as the Γ distribution function in the continuous variable t'. It is as if evolution is made up of random unitary time "events", where the probability density for $k = t/\tau_2$ events to take place in a time t' is given by Eq. (9). τ_2 is the average interval between two "events" and τ_1 is the time width of each event. Interpreted in the frame of the measurement theory τ_2^{-1} is the observation rate, τ_1 is the time width of each observation. The interaction of the system with a measuring apparatus is described by two intrinsic and model independent characteristic times: τ_1 and τ_2. In this description it is possible to obtain a dynamical definition of state reduction. The equation (4) written in the energy representation becomes

$$\bar{\rho}_{n,m}(t) = \frac{1}{(1 + i\omega_{n,m}\tau_1)^{t/\tau_2}} \rho_{n,m}(0) = e^{-\gamma_{n,m}t} e^{-i\nu_{n,m}t} \rho_{n,m}(0), \tag{10}$$

where

$$\gamma_{n,m} = (1/2\tau_2) ln(1 + \omega_{n,m}^2 \tau_1^2) \qquad \nu_{n,m} = (1/2\tau_2) tan^{-1}(\omega_{n,m}\tau_1) \tag{11}$$

When n = m, $\omega_{n,m} = 0$ and therefore $\bar{\rho}_{n,n} = \rho_{n,m}$, i.e. the energy is a constant of motion, while for $n \neq m |\bar{\rho}_{n,m}(t)| \to 0$ with a rate $\gamma_{n,m}$. Therefore $\bar{\rho}(t) \to \sum_n \rho_{n,n} |n\rangle\langle n|$ i.e. $\bar{\rho}$ approaches the diagonal form.

This theory is essentially stating that our perception of the world is a coarse-grain description of the real time evolution.

IV. DISCUSSION OF EXPERIMENTAL RESULTS

The idea of introducing a fundamental interval of time in quantum mechanics is not new. It has been pioneered in 1956 by Caldirola [9] to describe the quantum theory of electrons through a finite-difference Schrödinger equation. More recently some authors [10] have suggested that a discretization of time can solve many of the difficulties associated with the infinities in quantum field theory when dealing with particle interactions. The basic theme of the majority of these studies is that with a discrete model of space-time and quantum field theory, by allowing the discrete space and time interval to become small, both Minkowski space and conventional field theory emerge.

Assuming the existence of a "cronon" of time, it is important to determine the possibility of inferring its existence and eventually its value from a suitable experiment. Some experiments, irrespective of the problem of decoherence, have been thought in order to determine the existence of a fundamental unit of time in physics, but the experimental requirements are for the moment beyond the technological limits [10,11].

In order to test the validity of this theory Bonifacio has suggested some possible validations and has also compared his theory with recent experimental results [12,13]. In the experiment of Haroche's group [12] a system of two level atoms is injected one at a time in a high Q-resonant cavity and prepared in a way so that the atomic and cavity decay time and the decoherence time due to external coupling are very long. The atoms are injected in the excited state and oscillate between the upper and lower state so that the population difference oscillate as $\cos(\Omega \mathrm{t})$ where $\Omega = g\sqrt{n+1}$ is the Rabi frequency, n is the average number of photons in the cavity and g is the one photon Rabi frequency accordingly with the Jaynes-Cumming Hamiltonian model [14].

According to the formalism shown in Sec. III, these oscillations should be damped at a rate given by $\gamma = (1/2\tau_2) ln(1 + \Omega^2\tau_1^2)$ for the case of the electromagnetic field in the vacuum state. If $\gamma t >> 1$, where t is the time of flight of the atoms in the cavity, then the upper level population approaches the value 1/2. This damped behavior has indeed been observed in the Haroche's experiment. It seems that, even when all the dissipation mechanism appear to be ineffective, Rabi oscillation are damped in agreement with the above theoretical model.

In the experiment of Meekhof *et al.* [13] a similar type of damping has been observed and an increase of the damping constant has been found in agreement with the following curve $\gamma_n = \gamma_0(n = 1)^{0.7}$. Bonifacio's model can eventually fit this curve with a particular value of $g\tau_1$.

V. CONCLUSION

It is easy to see that both the theories analyzed in this paper reach the same conclusion: in any physical system decoherence has to be present. Zurek's model requires a certain number of assumptions and with the introduction of other approximations like perturbative expansion, complete positivity for the time evolution and Markov approximation [1] a ME of the Linblad form for the reduced density operator can be obtained. Bonifacio's theory, instead, does not introduce any further assumption, the model actually shows dynamically the Von Neumann state reduction. This means that this postulate can be dropped from those five postulate of quantum mechanics once non unitarity of the time evolution of the density operator is introduced. The fact that the two theories obtain the same results is not at all surprising since a ME of the Linblad form can be obtained everytime the evolution of the density operator has the semigroup property [1].

It is possible to say that the coupling with the environment seems sufficient to show decoherence, but it is not yet shown to be necessary. In this respect the proposed experiment [5] of Sec. II can be really a good test for determining the existence of a fundamental unit of time. Once the decoherent effects due to the coupling with the environment are "controlled" at least for an appreciably amount of time, then the existence of any residual decoherent effect can be eventually a signature of a "cronon" of time.

Intrinsic decoherence on the other hand could well explain the state reduction without any assumption, but this inevitably implies that if the evolution of the density operator is not continuous also the Schrödinger equation has to satisfy a finite difference equation. Therefore the elimination of the Von-Neumann reduction assumption seems to invoke also the change of the postulate regarding the existence of a Schrödinger equation continuous in the parameter t.

In conclusion what seems more convincing of the model explained in Sec. III, is that the introduction of a discretized stochastic evolution allows to determine at the microscopic level the dynamical origin of the mechanism of decoherence without specifically saying anything about the environment. In order to see the effect of a “cronon” of time, one way is to infer what are the possible physical situations in which it can be manifest through particular effects of a finite-difference Schrödinger equation, while absent from a timewise continuous one. Another possibility is to design an experiment in which all the system-environment decoherent coupling effects can be accurately estimated and verify eventually the presence of additional decoherences. Needless to say, both cases constitute a big challenge from the experimental point of view.

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