

Physics 566: Quantum Optics

UNM: Spring 2002, Prof. I.H. Deutsch

Lecture 1: Introduction

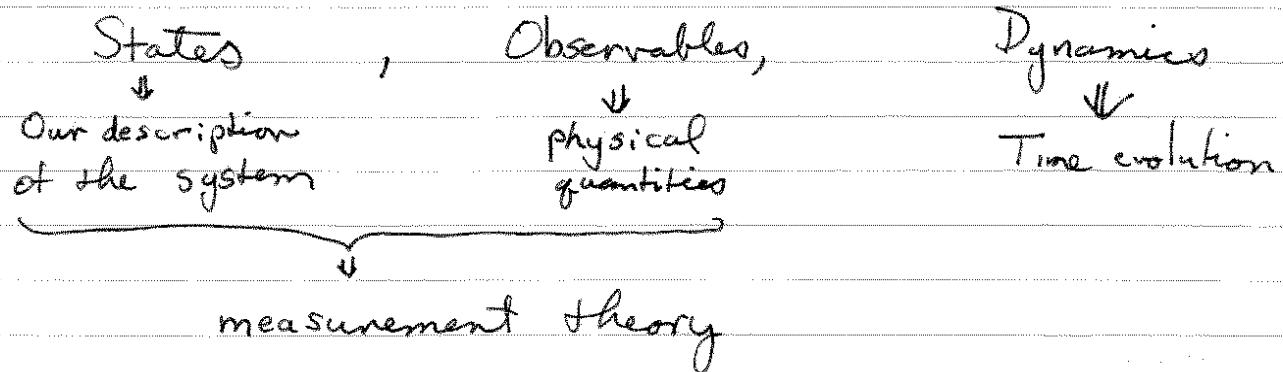
I Quantum Optics: The study, manipulation, and control of quantum mechanical coherence associated with optical (electromagnetic) fields

Two aspects of quantum physics:

- Wave - interference
- Particle - discrete events, "collapse", stochastic

History + Connections: See "Map of Q. Optics"

II. Review of Quantum Mechanics:



States: "Pure State": Best possible description of the system

$| \Psi \rangle$ Vector in Hilbert space

(complex inner-product vector space with possibly ∞ dimension)

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Observable: Hermitian operator $\hat{A} = \hat{A}^+$

Possible outcomes of measurements $\{a\}$

= eigenvalues $\hat{A}|a\rangle = a|a\rangle$

↑
real

Complete set $\sum_a |a\rangle \langle a| = \hat{I}$

\hat{P}_a : Projection operator $\hat{P}_a^2 = \hat{P}_a$

Probability of finding outcome a given state $|\Psi\rangle$

$$P(a|\Psi) = \frac{|\langle a|\Psi\rangle|^2}{\|\Psi\|^2}$$

Equivalence class: Ray in Hilbert space

Convention: restrict $\|\Psi\|^2 = \langle \Psi|\Psi \rangle = 1$

Overall phase of $|\Psi\rangle$ is not physical

Expected value of a measurement:

$$|\Psi\rangle = \sum_a |a\rangle \underbrace{\langle a|\Psi\rangle}_{= c_a} \quad (\text{Prob. amp})$$

$$|c_a|^2 = P(a|\Psi)$$

$$\Rightarrow \langle \hat{A} \rangle_\Psi = \sum_a a P(a|\Psi) = \sum_a a |\langle a|\Psi\rangle|^2$$

$$= \sum_a a \langle \Psi | a \rangle \langle a | \Psi \rangle$$

$$= \langle \Psi | \left(\sum_a a |a\rangle \langle a| \right) | \Psi \rangle$$

= $\langle \Psi | \hat{A} | \Psi \rangle$ matrix element

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Uncertainty: Variance

$$\Delta A^2 = \langle (\hat{A} - \langle \hat{A} \rangle)^2 \rangle = \langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2$$

Composable measurements commute

$$\Delta A \Delta B \geq \frac{1}{2} |\langle [\hat{A}, \hat{B}] \rangle|$$

Quintessential example $[\hat{x}, \hat{p}] = i\hbar$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$
 (minimum uncertainty
product is independent of state)

Another example:

$$[\hat{J}_x, \hat{J}_y] = i \hbar \hat{J}_z \quad (\text{units of } \hbar)$$

$$\Delta \hat{J}_x \Delta \hat{J}_y \geq \frac{1}{2} |\langle \hat{J}_z \rangle| \quad (\text{min uncertainty depends on state})$$

Measurement < Probability of outcome
post-measurement state

$$\begin{aligned} P(a|\Psi) &= |\langle a | \Psi \rangle|^2 = \langle \Psi | a \rangle \langle a | \Psi \rangle \\ &= \langle \Psi | \hat{P}_a | \Psi \rangle = \langle \hat{P}_a \rangle_{\Psi} \end{aligned}$$

$$\text{After measurement: } |\Psi' \rangle = \frac{\hat{P}_a |\Psi \rangle}{\sqrt{\langle \Psi | \hat{P}_a | \Psi \rangle}} = |a\rangle$$

Up to this point, I have implicitly assumed that the eigenvalues of \hat{A} were nondegenerate. In that case \hat{P}_a is

a one dimensional projector (axis in H-space)

"ODOP": One dimensional orthogonal projector $\hat{P}_a \hat{P}_a = \hat{P}_a$

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More generally, there will be degeneracies

$$\hat{A}|a,i\rangle = a|a,i\rangle$$

other q-numbers that specify pure state
"degeneracy" g_a

$$\hat{P}_a = \sum_{i=1}^{g_a} \hat{P}_{a,i} = \sum_i |a,i\rangle \langle a,i| \quad g_a\text{-dim } \text{orthogonal projector}$$

If measurement has no information about $\{c_i\}$

$$P(a|\Psi) = \langle \Psi | \hat{P}_a | \Psi \rangle = \sum_i |c_{a,i}|^2$$

$$|\Psi'\rangle = \frac{\hat{P}_a |\Psi\rangle}{\sqrt{\langle \Psi | \hat{P}_a | \Psi \rangle}} = \sum_i c_{a,i} |a,i\rangle$$

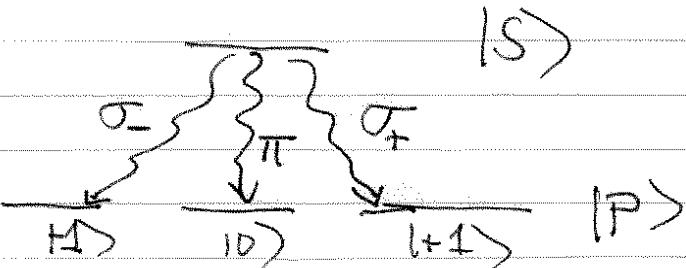
If we learn $\{c_i\}$

$$P(a,i|\Psi) = |c_{a,i}|^2$$

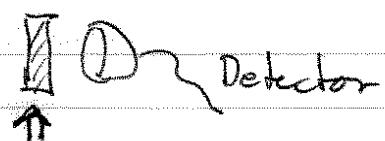
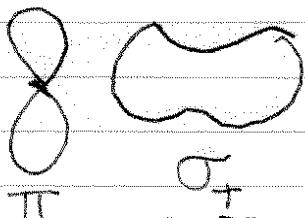
$$|\Psi'\rangle = |a,i\rangle$$

Example: Fluorescence

Decay of an atom



~~Dipole~~ Dipole patterns



Polarization σ_{\pm} filter

With filter \Rightarrow ODDP
w/o filter \Rightarrow Superposition state

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Quantum Superposition and Interference

Though the overall phase of the state has no physical meaning, the relative phase in a superposition is crucial:

$$|\Psi\rangle = c_1 |\phi_1\rangle + c_2 |\phi_2\rangle = |c_1| |\phi_1\rangle + e^{i\phi} |c_2| |\phi_2\rangle$$

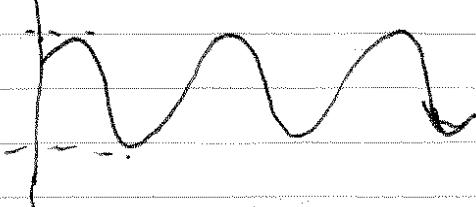
where $\phi = \text{Arg}(c_2) - \text{Arg}(c_1)$

$$\begin{aligned} P(a|\Psi) &= P(a|\phi_1) P(\phi_1) + P(a|\phi_2) P(\phi_2) \\ &\quad + 2 \underbrace{|c_1| |c_2| |\langle \phi_1 | a \rangle| |\langle \phi_2 | a \rangle| \cos(\phi + \delta)}_{\text{Interference!}} \end{aligned}$$

All that's strange and wonderful in quantum mech

$$P(a|\Psi) = A_1^2 + A_2^2 + 2 A_1 A_2 \cos(\phi + \delta)$$

$$A_i = |c_i \langle a | \phi_i \rangle| \quad \delta = \text{Arg} (\langle a | \phi_2 \rangle \langle \phi_1 | a \rangle)$$



Coherence
= Interference

$$\text{Visibility: } \frac{\max - \min}{\max + \min} = \frac{2 A_1 A_2}{A_1^2 + A_2^2}$$

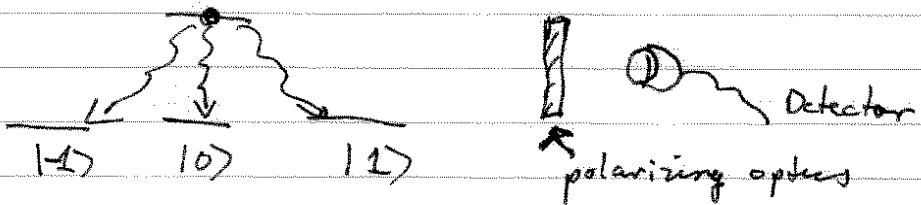
Distinguishability: If $\langle \phi_1 | a \rangle \langle a | \phi_2 \rangle = 0$

\Rightarrow No interference

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Density operator:

Suppose the experimenter prepares an atom



with 50% $|1\rangle$, 50% $|1\rangle$

Suppose the experimenter does not tell us whether she uses a right-handed or left-handed filter?

How do we calculate the probability of finding
 $|\phi\rangle = \alpha|1-1\rangle + \beta|1+1\rangle$?

Answer:

$$P_\phi = P(\phi|1-1) P(-1) + P(\phi|1+1) P(+1)$$

Statistical mixture: No interference

$$= \langle -1 | \hat{P}_\phi | -1 \rangle \left(\frac{1}{2}\right) + \langle +1 | \hat{P}_\phi | +1 \rangle \left(\frac{1}{2}\right)$$

$$= \langle \hat{P}_\phi \rangle$$

The state of a statistical mixture cannot be written as a vector in Hilbert space.

More general state: "Density operator"

$$\hat{\rho} = \sum_{\psi} P_{\psi} |\psi\rangle \langle \psi|$$

$$\begin{aligned} P_{\psi} &\geq 0 \\ \sum P_{\psi} &= 1 \end{aligned} \quad \left. \begin{array}{l} \text{Probabilities} \\ \hline \end{array} \right.$$

Statistical mixture of pure states

Trace operation: $\text{Tr}(\hat{A}) = \sum_{n=1}^{\dim H} \langle n | \hat{A} | n \rangle$

$$\text{Tr}(1\psi \times \psi | \hat{A}) = \langle \psi | \hat{A} | \psi \rangle = \langle \hat{A} \rangle$$

More generally: $\langle \hat{A} \rangle = \text{Tr}(\hat{A} \hat{\rho})$

$$= \sum_{\psi} P_{\psi} \langle \psi | \hat{A} | \psi \rangle$$

$$\Rightarrow P_{\psi} = \text{Tr}(\hat{P}_{\psi} \hat{\rho}) = \sum_{\psi} P_{\psi} \langle \psi | \hat{P}_{\psi} | \psi \rangle$$

Properties of density operator

- Hermitian $\hat{\rho} = \hat{\rho}^+$
- Normalized $\sum_{\psi} P_{\psi} = \text{Tr}(\hat{\rho}) = 1$
- "Positive" \Rightarrow All eigenvalues $\lambda \geq 0$
- Pure state $\Rightarrow \hat{\rho} = |\Psi\rangle \langle \Psi|$ for some $|\Psi\rangle$

In another basis $\hat{\rho} = \sum_{n,m} P_{nm} |n\rangle \langle m|$

- Diagonal elements $\langle n | \hat{\rho} | n \rangle$ = "Populations" in $|n\rangle$

- Off-diagonal elements $\langle n | \hat{\rho} | m \rangle$ = "Coherences"

Pure state $\text{Tr}(\hat{\rho}^2) = 1$

Mixed state $\text{Tr}(\hat{\rho}^2) < 1$