Physics 566 Spring 2002

Lecture 5: Optical Bloch Equation (II)
Phenomenological Damping

So far we have considered only "coherent evolution". That is, we have considered the dynamics of of two-level systems governed by a simple time dependent Hamiltonian. The resulting evolution exhibited a simple sinusoidal time dependence for all time — it is reversable. In real situations this is never the case — the systems eventually "relax" to some steady state where "memory" of the initial condition is lost. this irreversible behavior is a fact of nature. How do we take irreversibility into account in quantum theory whose basic time evolution is a reversible unitary map? This is a fundamental problem which we will treat in detail in the latter 1/3 of this class. For now we will take a somewhat phenomenological approach. As an aside, the problem of obtaining effective irreversible behavior at the macroscopic level when the microscopic laws of physics are time-reversible is a fundamental issue in classical physics as well; i.e. from Newtonian dynamics to the second law of thermo.

Phenomenological decay:

Consider a two-level system. There are two kinds of decays

- Relaxations of the population ρ_{ee}, ρ_{gg}
- Relaxations of the coherences ρ_{eg}

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In the old NMR transition:

1) $T_1 \equiv$ decay time constant of the population inversion $w \equiv \rho_{ee} - \rho_{gg}$
$\equiv$ "Longitudinal relaxation"

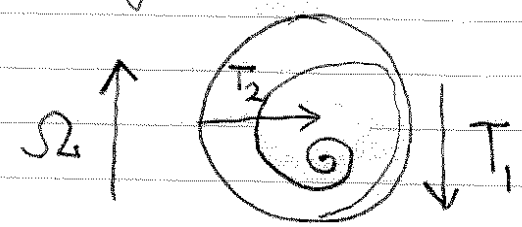
2) $T_2 \equiv$ decay time constant of coherences
$\equiv$ "Transverse relation"

Block equations with relaxation

$$\dot{u} = -\Delta v - \frac{1}{T_2} u$$
$$\dot{v} = \Delta u + \Omega w - \frac{1}{T_2} v$$
$$\dot{w} = -\frac{1}{T_1}(w - w_{eq}) - \Omega v$$

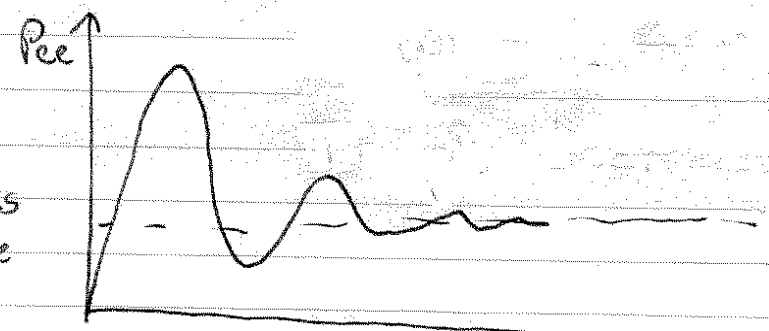
w_{eq} = equilibrium inversion $= -1$

Trajectory on Bloch sphere



State is no longer described by a pure state. Must use density op.

Damped Rabi oscillations



Another picture of "T_1" vs. "T_2"

Consider $\hat{\rho} = \sum_\alpha P_\alpha |\psi_\alpha\rangle\langle\psi_\alpha|$ (ensemble decomposition)

$$\rho_{ij} = \sum_\alpha P_\alpha c_i^\alpha c_j^{\alpha *} = \sum_\alpha P_\alpha |c_i^\alpha||c_j^\alpha| e^{i(\phi_i^\alpha - \phi_j^\alpha)}$$

Diagonal element $\rho_{ii} = \sum_\alpha P_\alpha |c_i^\alpha|^2$

$\Rightarrow$ T_1 involves average relaxation of (magnitude)^2 of probability amplitudes

$$\rho_{ii}(t)|_{relax} = e^{-\Gamma_i t}\rho_{ii}(0)$$

Γ_i = decay rate of $\overline{|c^\alpha|^2} \Rightarrow \overline{|c_i^\alpha(t)|} \approx \overline{|c_i^\alpha(0)|}\, e^{-\frac{\Gamma_i}{2}t}$

Off-diagonal, Suppose magnitude same $\forall \alpha$

$$\Rightarrow \rho_{ij}(t) = |c_i(0)||c_j(0)|\, e^{-\left(\frac{\Gamma_i+\Gamma_j}{2}\right)t} \sum_\alpha P_\alpha e^{i(\phi_j^\alpha - \phi_j^\alpha)}$$

Consider process of "dephasing" $\Rightarrow \phi_j^\alpha - \phi_j^\alpha = (\delta\omega^\alpha)t$

$$\sum_\alpha P_\alpha e^{i(\delta\omega)t} \Rightarrow e^{-\beta_{ij}t}$$

$\Rightarrow$ T_2 involves both population change and dephasing

$$\boxed{\frac{1}{T_2} = \frac{\Gamma_i + \Gamma_j}{2} + \beta_{ij} \equiv \gamma_{ij}}$$

Sources of relaxation – Perturbation by the "environment"

e.g. Collisions – elastic ⇒ dephasing
inelastic ⇒ population change

Generally $T_2 \ll T_1$

Small changes in phase compared to pop. change.

Note: If we have an ensemble of system dephasing can occur due to inhomogeneities (see H.W.). Then $\frac{1}{T_2} = \frac{1}{T_2'} + \frac{1}{T_2^*}$

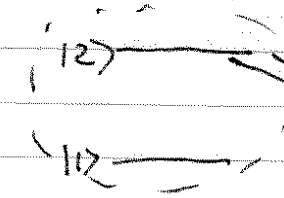
where T_2^* = decay time due to inhomogeneity

T_2' = decay " for homogeneous sample (truly irreversible)

Unless otherwise stated, we will take $T_2^* = \infty$

Non unitary Schrödinger equation

Consider system with loss of population to extra level which we do not account for



In $\{|1\rangle, |2\rangle\}$ space $\mathrm{Tr}(\rho)$ not conserved

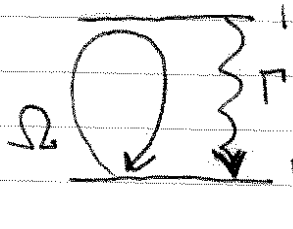
⇒ Simple approx: Deterministic but non unitary

$$\begin{cases} \dot{C}_1 = -i\frac{E_1}{\hbar} C_1 - i\frac{H_{12}}{\hbar} C_2 \\ \dot{C}_2 = -i\frac{E_2}{\hbar} C_2 - \frac{\Gamma}{2} C_2 - i\frac{H_{21}}{\hbar} C_1 \end{cases}$$

Decaying level ⇒ imaginary part of energy

Master equation for two-level atom

Consider a two-level "closed transition"



One source of "relaxation": Spontaneous emission

Γ = "Einstein A-coefficient" $= \frac{1}{\tau} = \frac{4}{3}\frac{|\vec{d}_{ge}|^2}{\hbar} k^3$

τ = Natural lifetime

$$\dot{\rho}_{ee}|_{relax} = -\Gamma \rho_{ee} = -\frac{1}{T_1}\rho_{ee} \qquad T_1 = \frac{1}{\Gamma} = \tau$$

$$\dot{\rho}_{eg}|_{relax} = -\frac{\Gamma}{2}\rho_{eg} = -\frac{1}{T_2}\rho_{eg} \qquad T_2 = \frac{T_1}{2}$$

Since $\rho_{ee} + \rho_{gg} = 1$ $\quad \dot{\rho}_{gg}|_{relax} = -\dot{\rho}_{ee}|_{relax} = +\Gamma\rho_{ee}$

Master equation for damped Rabi flopping

$$\dot{\rho}_{ee} = -\Gamma\rho_{ee} + i\frac{\Omega}{2}(\rho_{ge} - \rho_{eg}) = -\dot{\rho}_{gg}$$

$$\dot{\rho}_{ge} = \left(-i\Delta - \frac{\Gamma}{2}\right)\rho_{ge} - i\frac{\Omega}{2}(\rho_{ee} - \rho_{gg})$$

↖ Same as Block e.q.

Operator form:

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] + \mathcal{L}_{relax}[\hat{\rho}]$$

$$\mathcal{L}_{relax}[\hat{\rho}] = \underbrace{-\frac{\Gamma}{2}(\hat{\sigma}_+\hat{\sigma}_-\hat{\rho} + \rho\hat{\sigma}_+\hat{\sigma}_-)}_{\text{Decay}} + \underbrace{\Gamma\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+}_{\text{replenish.}}$$

Steady-state solutions set $\dot{\rho} = 0$

$$\Rightarrow \rho_{ge}^{(s.s.)} = \left(\frac{-i\Omega/2}{i\Delta + \Gamma/2}\right)\underbrace{(\rho_{ee}^{s.s.} - \rho_{gg}^{s.s.})}_{W^{s.s.}} = \frac{1}{2}(u^{s.s.} + iv^{s.s.})$$

$$\rho_{ee}^{(s.s.)} = i\frac{\Omega}{2\Gamma}(\rho_{ge}^{(s.s.)} - \rho_{eg}^{(s.s.)}) = -\frac{\Omega}{\Gamma}\mathrm{Im}(\rho_{ge}^{s.s.})$$

$$= -\frac{\Omega^2}{\Delta^2 + \frac{\Gamma^2}{4}}(\rho_{ee}^{s.s.} - \rho_{gg}^{s.s.}) = \frac{-\frac{\Omega^2}{4}}{\Delta^2 + \frac{\Gamma^2}{4}}(2\rho_{ee} - 1)$$

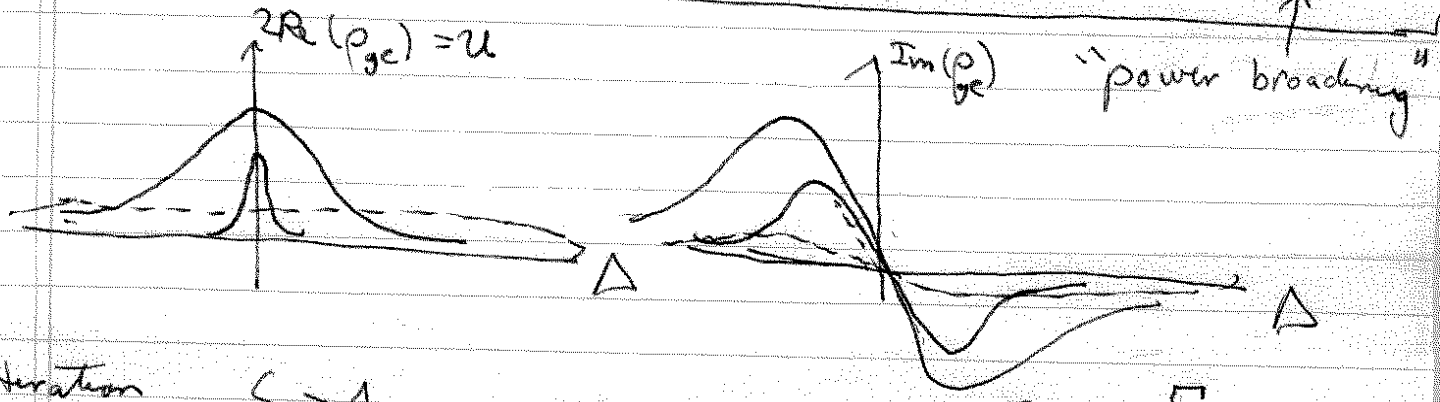
Define $S \equiv \dfrac{\Omega^2/2}{\Delta^2 + \Gamma^2/4}$ = "Saturation parameter"

$$\Rightarrow \rho_{ee}^{s.s.} = -\frac{S}{2}(\rho_{ee}^{s.s.} - 1) = -S\rho_{ee}^{s.s.} + S/2$$

$$\Rightarrow \boxed{\rho_{ee}^{s.s.} = \frac{S/2}{1+S}, \quad \rho_{gg}^{s.s} = \frac{1+S/2}{1+S}, \quad W^{s.s} = \frac{-1}{1+S}}$$

$$\Rightarrow \boxed{\rho_{ge}^{s.s.} = \frac{\Omega/2}{+\Delta + i\frac{\Gamma}{2}}\left(\frac{-1}{1+S}\right) = \left(-\Delta + i\frac{\Gamma}{2}\right)\frac{\Omega/2}{\Delta^2 + \frac{\Gamma^2}{4}\left(1 + \frac{2\Omega^2}{\Gamma^2}\right)}}$$

"power broadening"



Saturation intensity $S = 1$ on resonance $\Rightarrow \Omega = \dfrac{\Gamma}{\sqrt{2}}$

$S = I/I_{sat}$