Lecture 14: Nonclassical light Continued

Last lecture we saw two nonclassical phenomena

- Photon anti-bunching \( g^{(2)}(0) < g^{(2)} \)
- Sub-Poissonian statistics

\[
g^{(3)}(0) - 1 = \frac{\langle \hat{a}^{+} \hat{a} \hat{a}^{+} \hat{a} \rangle}{\langle \hat{a}^{+} \hat{a} \rangle^2} - 1 = \frac{\langle n^2 \rangle - \langle n \rangle}{\langle n \rangle^2} < 0
\]

These are closely related but not identical. There are phenomena which exhibit one but not the other.

First observation of photon anti-bunching:

Resonance Fluorescence

Detector

atom \( \rightarrow \) exciting laser


\[ C(\tau) \] Combinant rate

\[ C(0) < C(\tau) \] (ns)
Why is resonance fluorescence anti-bunched?

Detection of first photon projects atom into ground state (quantum measurement). Emission of the next photon takes some time, absorption followed by spontaneous emission again.

Scattering rate: \( \Gamma' = N_e^S \cdot \Gamma \)  

\[ N_e^S = \frac{s}{2} \text{ (for weak excitation)} \]

\[ s = \frac{\Delta^2}{\Delta^2 + \Gamma_0^2} \]

The Kimble et al. experiment did not show sub-Poissonian statistics. The reason is that the atomic beam has fluctuations. We must properly convolve atomic statistics with photon statistics.

Later experiment: Single trapped ion

(14. Walther PRL 58 203 (1987))

\[ g^{(2)}(\tau) \]

-20 0 20
\[ \tau (\text{ns}) \]

Shows both anti-bunching and sub-Poissonian stats.
Other sources of nonclassical light

A spectral cascade

Starting in the mid 1980's there began strong interest in the nonclassical features of light produced in nonlinear optical crystals.

Parametric down conversion

The pump and signal can beat together to produce electric polarization that oscillates at $\omega_p - \omega_s$

$$P = \chi^{(2)} E_p E^*_s = \chi^{(2)} E_p E^*_s e^{i(k_p - k_s) \cdot x} e^{-i(\omega_p - \omega_s) t}$$

Interaction energy

$$H = P E^*_i + P^* E_i = \chi^{(2)} E_p E^*_s E_i e^{i(k_s \cdot x - \Delta\omega t)} + c.c.$$
For long crystals, long interaction times, interaction is strong if

\[ \Delta k = 0 \Rightarrow \mathbf{k}_p = \mathbf{k}_s + \mathbf{k}_i \]  
\[ \Delta \omega = 0 \Rightarrow \omega_p = \omega_s + \omega_i \]  
matching

Note: In order to achieve phase matching, we require a birefringent crystal. This constrains a relation between the pump, signal, and idler field polarization vectors.

Parametric Fluorescence

Suppose no signal field is injected into the crystal. We will still get subharmonic generation due to spontaneous downconversion.

The spontaneous process is a quantum phenomenon. The pump photon can, through the nonlinearity, decay into two daughter photons. The phase matching condition can be interpreted as energy and momentum conservation.

\[ h \omega_p = h \omega_s + h \omega_i \]  
\[ \pm \mathbf{k}_p = \pm \mathbf{k}_s + \pm \mathbf{k}_i \]  
(uncertainty principle)
Because there is not injected prepared signal field there is a continuum of possible pairs that can be generated. The photons are born on complementary cones to satisfy phase matching:

\[ \mathbf{k}_s + \mathbf{k}_i = \mathbf{k}_p \]

\[ k = n(\omega) \frac{\omega}{c} \]

\[ \omega_p = \omega_i + \omega_s \]

\[ \leftarrow \text{cones of constant color} \]

Head on view

\[ \leftarrow \text{signal and idler on conjugate cones} \]

Degenerate PARAM \( \omega_s = \omega_i = \frac{\omega_p}{2} \)

Phase matching determines the directions of output.
Quantum state description

Let us assume that the pump is a strong classical field. The down-conversion process is not very efficient, so we can further assume that the pump is not changed very much (undephased). The signal and idler fields are weak and must be quantized.

\[ \hat{H} = \int (\chi^{(2)} E_p^* E_s E_c^* + \chi^{(2)} E_s^* E_s E_c) \ d^3x \]

\[ = \int \sum_{b_i, k_s} \left( \frac{\chi^{(2)}}{h_{ji}} a_s^j a_i^j + \frac{\chi^{(2)*}}{h_{si}} a_s^j a_i^j \right) \]

\[ \chi^{(2)} = \chi^{12} E_p^* E_{vac}^2 \]

The Hamiltonian thus involves creation and annihilation of pairs of photons. Because there is no preference for a given pair, the state produced, to lowest order perturbation

\[ |\psi\rangle = |\text{Vacuum}\rangle + \int d^3k_s 1_{k_s} \langle 0 | 1_{k_p-k_s} \rangle \]

Entangled state (more later)
Properties of correlated photons

- Photon pair born almost simultaneously (fast off-resonant process) \( \sim \) femtoseconds

- Time of birth of given pair is uncertain \( \sim T_p \) the coherence time of pump

\[ \Delta \omega = \omega_s - \omega_i \sim \delta \nu_p \]

And in the frequency domain, the difference in frequency between the signal and idler is very small \( \Delta \omega_s = \omega_s - \omega_i \sim \delta \nu_p \) the pump laser linewidth. On the other hand, the individual s and i photons are broad (short pulse wave packets). Note the pump is CW, yet we see short fast phenomenon.

\[ C(t) \sim \]

Cerenkov experiment

Correlation

[Diagram: \( \chi^{(2)} \) with branching and 0 and 1 transitions]
Two-photon Interferometry

Photon pairs are sent to unbalanced interferometer, and counted in "singles" and "coincidence".

FIG. 2. Two-photon interference in the dual-beam Michelson interferometer: The coincidence count rate (left axis, triangles in lower trace) and the singles count rate (right axis, squares in upper trace) vs the arm length difference for the setup of Fig. 1. The solid line denotes the theoretical prediction (see text). These data points were taken in a region far away from the white-light fringe (with $\Delta L \approx 240 \mu m > \Delta L_m = 50 \mu m$). The integration time per step was 1 sec.

The data shows no fringes in singles rate but an oscillation in the coincidence rate.

This is an example of two-photon interference.

P.G. Kwiat et al. PRA 29,10 (1984)
Recall Dirac's statement: "The photon only interferes with itself." This should only be applied when taking about first-order interference.

In quantum theory, indistinguishable processes interfere.

In our case:

Process A: Both photons take short path

Process B: Both photons take long

These processes are in principle indistinguishable =⇒ the interference.

Coincidence rate: \( G^{(2)}(1, 2) = \langle E_{(1)}^{(-)} E_{(2)}^{(+)} E_{(1)}^{(+)} E_{(2)}^{(-)} \rangle \)

\[
E_{(x,t)}^{(+)} = \frac{1}{2} \left( E_{(S,t)}^{(+)} + E_{(L,t)}^{(+)} \right)
\]

\[
\Rightarrow G^{(2)}(1, 2) = \frac{1}{4} \langle \Psi(1, 2) | \Psi(1, 2) \rangle
\]

\( \Psi(1, 2) = \frac{1}{4} \langle \Psi_{E(1)}^{(+)} \Psi_{E(2)}^{(+)} | \Psi \rangle \) two-photon wavefunction.
\[ \Phi(1,2) = \Phi(s, t_1; s, t_2) + 2\Phi(l, t_1; l, t_2) \]
\[ + 2\Phi(l, t_1; s, t_2) + 2\Phi(l, t_1; l, t_2) \]

\[ \Rightarrow G^{(2)}(1,2) = \left| \Phi(s, t_1; s, t_2) \right|^2 + \left| \Phi(l, t_1; l, t_2) \right|^2 \]

Invariance + \[ 2 \text{Re} \left( \Phi(s, t_1; s, t_2) \star \Phi(l, t_1; l, t_2) \right) \]

For narrow coincidence window only S-S and L-L term contribute

\[ \Rightarrow G^{(2)}(1,2) = 2 \left( 1 + \cos k_0 \Delta L \right) \frac{I(1) I(2)}{I(1) I(2)} \]

Note this theory predicts perfect visibility. The experiment, on the other hand, shows 50% visibility. This is because the coincidence window was too broad. So, does the experiment real exhibit nonclassical features?

Classical model: Random but correlated fields

\[ \vec{k}_c + \vec{k}_s = \vec{k}_p \]

\[ \vec{k}_c, \vec{k}_s \text{ random} \]

\[ P(\vec{k}_c, \vec{k}_s) = S(\vec{k}_p - (\vec{k}_c + \vec{k}_s)) \]
Many future experiments showed higher
visibility and violated classical model
of visibility.

50% visibility is consistent with classical model.

But

\( G(2) = \left< 1 + \cos k_{a} \lambda L \right\rangle \)

\( G(2) \approx \left< 1 + \cos k_{a} \lambda L \right\rangle (1 + \cos k_{s} \lambda L) \)

\( G(2) = \left< 1 + \cos k_{a} \lambda L + \cos k_{s} \lambda L \right\rangle + \frac{1}{2} \cos (k_{s} - k_{a}) \lambda L \)

\( \left< 1 + \cos k_{a} \lambda L \right\rangle = 1 \)