

Lecture 14: Nonclassical light Continued

Last lecture we saw two nonclassical phenomena

- Photon anti-bunching $g^{(2)}(0) < g^{(2)}(\tau)$

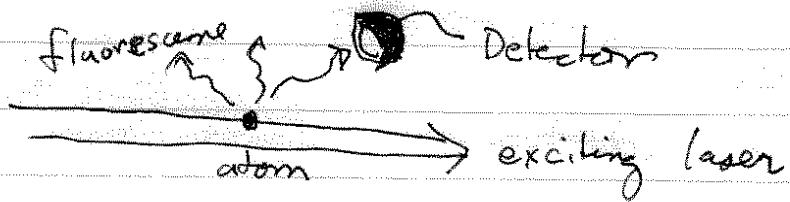
- Subpoissonian statistics

$$g^{(2)}(0) - 1 = \frac{\langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle}{\langle \hat{a}^\dagger \hat{a} \rangle^2} - 1 = \frac{\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2} < 0$$

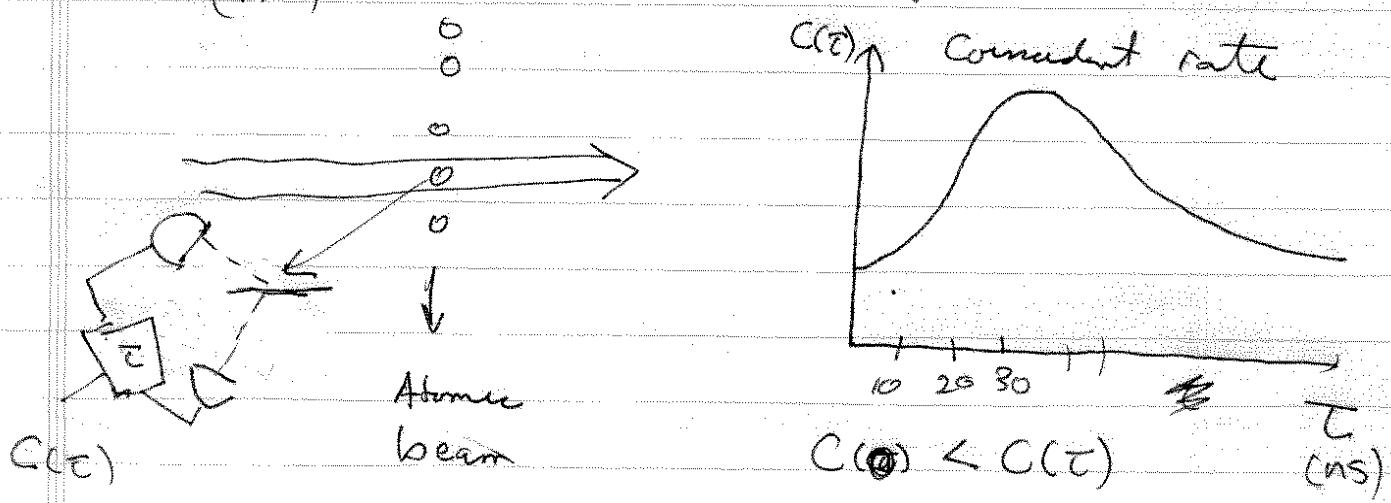
These are closely related but not identical.
There are phenomena which exhibit one but not the other.

First observation of photon anti-bunching:

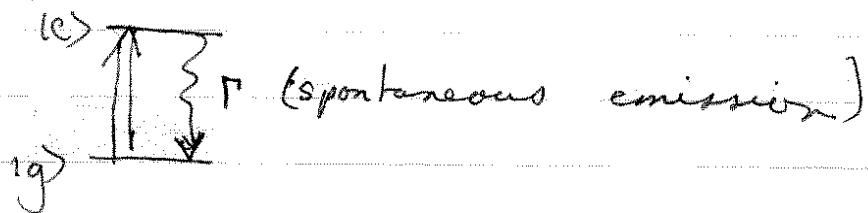
Resonance Fluorescence



Experiment H. J. Kimble, M. Dagenais, and L. Mandel
(1977)



Why is resonance fluorescence anti-bunched?



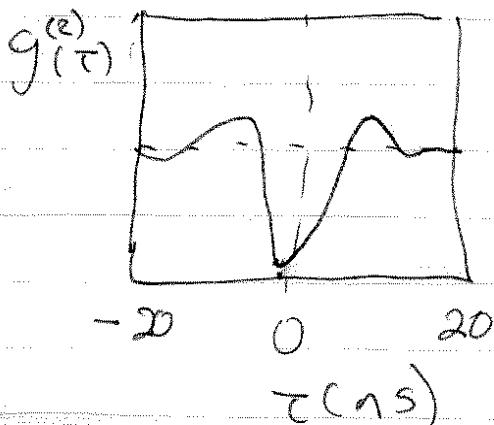
Detection of first photon projects atom into ground state (quantum measurement). Emission of the next photon takes some time, absorption followed by spontaneous emission again.

Scattering rate: $\Gamma' = N_e^{ss} \Gamma$ $N_e^{ss.}$ = steady state population on |e>

$$N_e^{ss.} = \frac{S}{2} \text{ (for weak excitation)} \quad S = \frac{\Delta^2 + \Gamma^2}{4}$$

The Kimble et al experiment did not show sub-poissonian statistics. The reason is that the atomic beam has fluctuations. We must properly convolve atomic statistics with photon statistics.

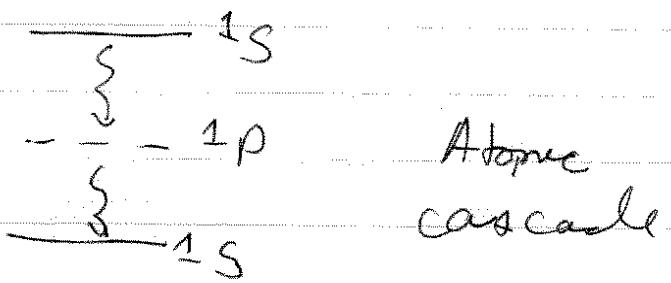
Later experiment: Single trapped ion
(H. Walther PRL 58 203 (1987))



Shows both anti-bunching and sub-poissonian stats.

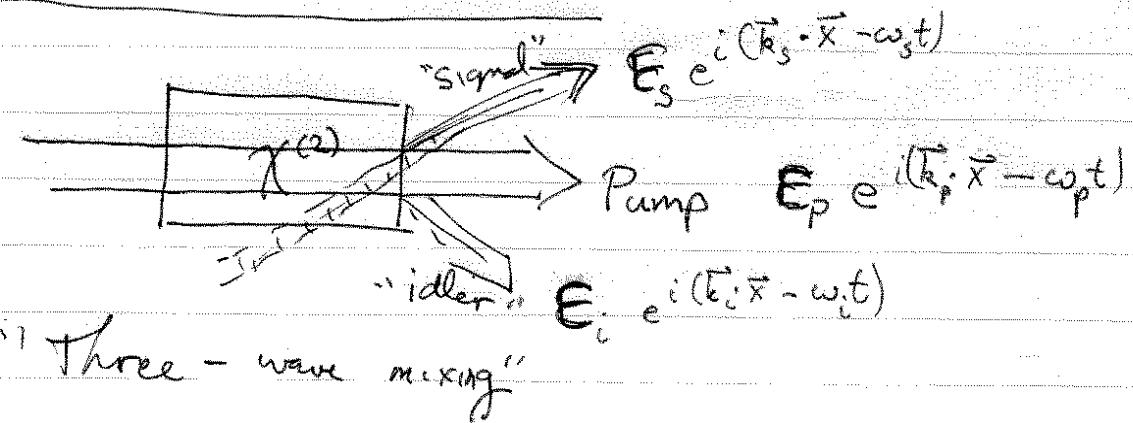
Other sources of nonclassical light

Aspect:



Starting in the mid 1980's there began strong interest in the nonclassical features of light produced in nonlinear optical crystals:

Parametric Downconversion



The pump and signal can beat together to produce of electric polarization that oscillated at $\omega_p - \omega_s$

$$\text{Power } P = \chi^{(2)} E_p E_s^* = \chi^{(2)} \epsilon_p \epsilon_s^* e^{i(\vec{k}_p - \vec{k}_s) \cdot \vec{x}} e^{-i(\omega_p - \omega_s)t}$$

Interaction energy

$$\begin{aligned} H &= P E_i^* + P^* E_i \\ &= \chi^{(2)} \epsilon_p \epsilon_s^* \epsilon_i e^{i(\vec{k}_p \cdot \vec{x} - \omega_s t)} + \text{c.c.} \end{aligned}$$

For long crystals, long interaction time, interaction is strong if

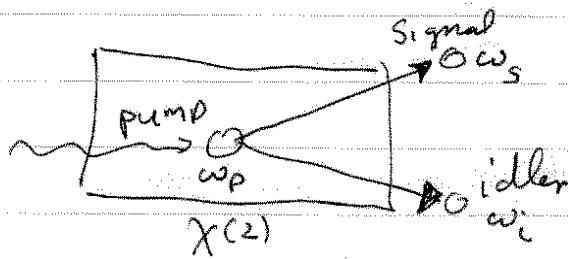
$$\Delta \vec{k} = 0 \Rightarrow \vec{k}_p = \vec{k}_s + \vec{k}_i \quad \text{Phase matching}$$

$$\Delta\omega = 0 \Rightarrow \omega_p = \omega_s + \omega_i \quad \text{frequency}$$

Note: In order to achieve phase matching we require a birefringent crystal. This constrains a relation between the pump, signal, and idler field polarization vectors.

Parametric Fluorescence

Suppose no signal field is injected into the crystal. We ^{will} still get subharmonic generation due to spontaneous downconversion



The spontaneous process is a quantum phenomenon. The pump photon can, through the nonlinearity, decay \rightarrow into two daughter photons. The phase matching condition can be interpreted as energy and momentum conservation

$$\hbar \omega_p = \hbar \omega_s + \hbar \omega_i \quad (\text{No within uncertainty principle})$$

$$\hbar \vec{k}_p = \hbar \vec{k}_s + \hbar \vec{k}_i$$

Because there is not injected preferred signal field there is a continuum of possible pairs that can be generated. The photons are born on complementary cones to satisfy phase matching.

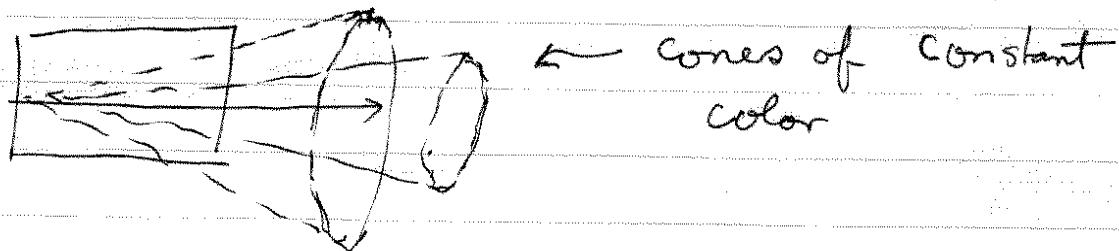
$$\vec{k}_s \quad \vec{k}_i$$

$\angle \theta$

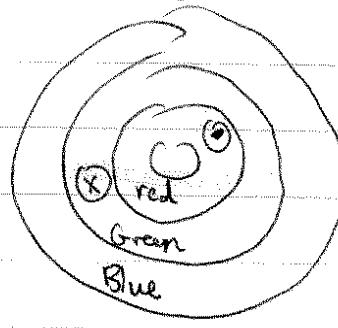
$$\vec{R}_p$$

$$k = n(\omega) \frac{\omega}{c}$$

$$\omega_p = \omega_i + \omega_s$$



Head on view



\leftarrow Signal and idler on conjugate cones

Degenerate PARAM $\omega_s = \omega_i = \frac{\omega_p}{2}$

Phase matching determines the directions of output.

Quantum state description

Let us assume that the pump is a strong classical field. The down-conversion process is not very efficient, so we can further assume that the ~~field~~ pump is not changed very much (unpleted). The signal and idler field are weak and must be quantized.

$$\rightarrow \hat{H} = \left(\chi^{(2)} E_p^* \hat{E}_s^{(+)} \hat{E}_i^{(+)} + \chi^{(2)} E_p \hat{E}_s^{(-)} \hat{E}_i^{(-)} \right) d^3x$$

crystal

$$= \sum_{k_s k_i} (\tilde{\chi}^{(2)} \hat{a}_s \hat{a}_i + \tilde{\chi}^{(2)*} \hat{a}_s^\dagger \hat{a}_i^\dagger)$$

(phase matched)

$$\tilde{\chi}^{(2)} = \chi^{(2)} E_p^* E_{\text{vac}}^2$$

(coupling constant)

The Hamiltonian thus involves creation and annihilation of pairs of photons. Because there is no preference for a given pair, the state produce, to lowest order perturbation

$$|n\rangle_{\text{si}} = |\text{Vacuum}\rangle + \int d^3k_s |1_{k_s}\rangle \otimes |1_{k_p - k_s}\rangle$$

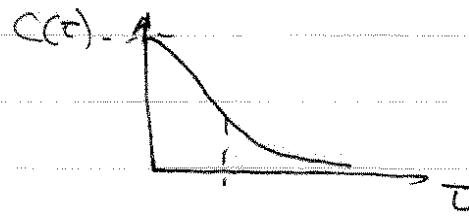
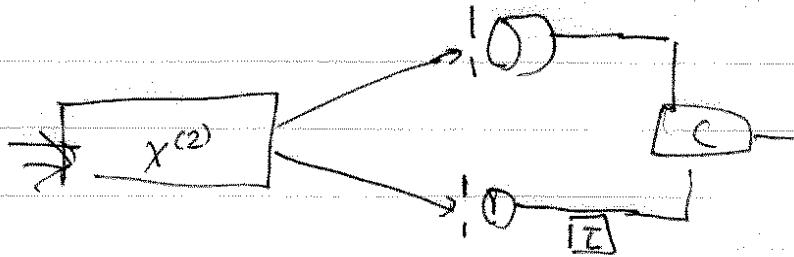
phase matched

Entangled State (more later)

Properties of correlated photons

- Photon pair born almost simultaneously (fast off resonant process) \sim femtoseconds
- Time of birth of given pair is uncertain \sim tip the coherence time of pump

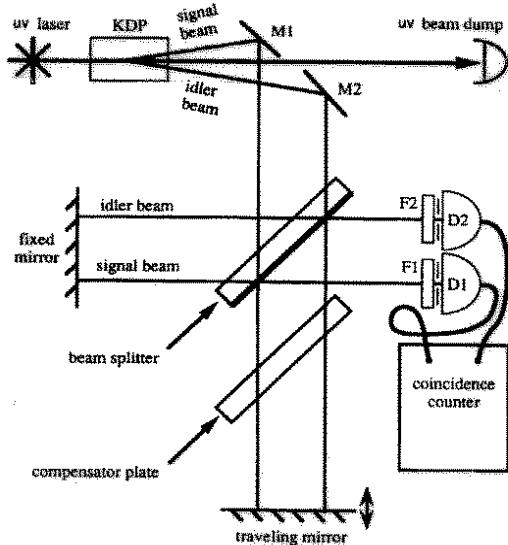
And in the frequency domain, the difference in frequency between the signal and idler is very small $\Delta\omega_s = \omega_s - \omega_i \sim \Delta\omega_p$, the pump laser linewidth. On the other hand, the individual s and i photons are broad (short pulse wave packet). Note the pump is CW, yet we see short fast phenomenon



Correlation
time

Cedarken
experiment

Two-photon Interferometry



P.G. Kwiat et al. PRA 29/0 (1990)

Photon pairs
are sent to unbalanced
interferometer, and
Counted in "singles"
and "coincidence"

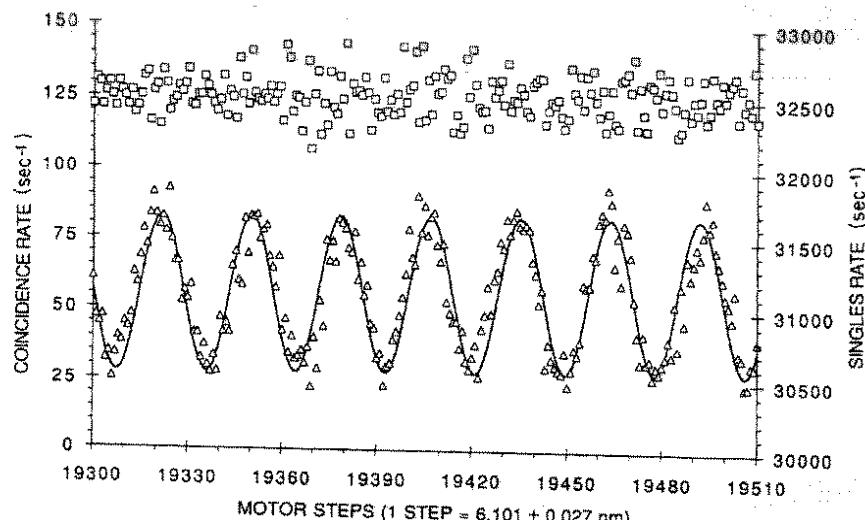


FIG. 2. Two-photon interference in the dual-beam Michelson interferometer: The coincidence count rate (left axis, triangles in lower trace) and the singles count rate (right axis, squares in upper trace) vs the arm length difference for the setup of Fig. 1. The solid line denotes the theoretical prediction (see text). These data points were taken in a region far away from the white-light fringe (with $\Delta L = 240 \mu\text{m} \gg \Delta l_s = 50 \mu\text{m}$). The integration time per step was 1 sec.

The data shows no fringes in singles rate but an oscillation in the coincidence rate.

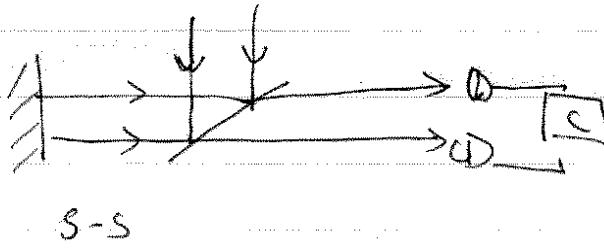
This is an example of two photon interference.

Recall Dirac's statement: "The photon only interferes with itself". This should only be applied when taking about first-order interference.

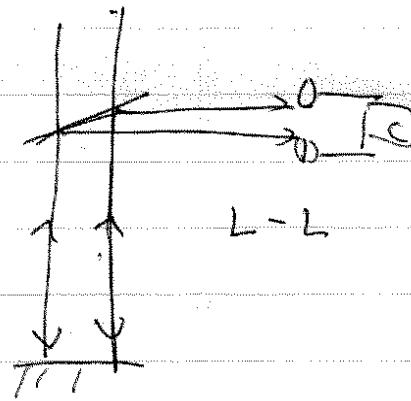
In quantum theory, indistinguishable processes interfere.

In out case

Process A: Both photons take short path



Process B: Both photon take long path



These process are in principle indistinguishable \Rightarrow they interfere.

Coincidence rate: $G^{(2)}(1,2) = \langle E_{(1)}^{(+)} E_{(2)}^{(+)} | E_{(1)}^{(+)} E_{(2)}^{(+)} \rangle$

$$E^{(+)}(\vec{x}, t) = \frac{1}{2} (E^{(+)}(S, t) \underset{\text{short}}{\uparrow} + E^{(+)}(L, t) \underset{\text{long}}{\uparrow})$$

$$\Rightarrow G^{(2)}(1,2) = \psi^*(1,2) \psi(1,2)$$

$$\psi(1,2) = \frac{1}{\sqrt{2}} \langle 0 | E_{(1)}^{(+)} E_{(2)}^{(+)} | \psi \rangle$$

two-photon wavefunction

K4.10

$$\Rightarrow \Psi(1,2) = \Psi(s_1 t_1; s_2 t_2) + \theta \Psi(s_1 t_1; L_2 t_2) \\ + \Psi(L_1 t_1; s_2 t_2) + \Psi(L_1 t_1; L_2 t_2)$$

$$\Rightarrow G^{(2)}(1,2) = |\Psi(s_1 t_1; s_2 t_2)|^2 + |\Psi(L_1 t_1; L_2 t_2)|^2$$

Interference $\rightarrow + 2 \operatorname{Re} (\Psi(s_1 t_1; s_2 t_2) \Psi^*(L_1 t_1; L_2 t_2))$

For narrow coincidence window only S-S and L-L term contribute

$$\Rightarrow \boxed{\frac{G^{(2)}(1,2)}{I(1) I(2)} = 2(1 + \cos k_p \Delta L)}$$

Note this theory predicts perfect visibility. The experiment, on the other hand, shows 90% visibility. This is because the coincidence window was too broad. So, does the experiment real exhibit nonclassical features?

Classical model: Random but correlated photons fields

$$\vec{k}_c + \vec{k}_s = \vec{k}_p$$

\vec{k}_c, \vec{k}_s random

$$P(\vec{k}_c, \vec{k}_s) = S(\vec{k}_p - (\vec{k}_c + \vec{k}_s))$$

$$\Rightarrow G_{\text{signal}}^{(1)} \propto \langle 1 + \cos k_s \Delta L \rangle = 1$$

$$G_{\text{idler}}^{(1)} \propto \langle 1 + \cos k_i \Delta L \rangle = 1$$

$$G^{(2)} \propto \langle (1 + \cos k_s \Delta L) (1 + \cos k_i \Delta L) \rangle$$

$$= \langle 1 + \cos k_s \Delta L + \cos k_i \Delta L$$

$$+ \frac{1}{2} \cos(k_s - k_i) \Delta L + \frac{1}{2} \cos(k_s + k_i) \Delta L \rangle$$

But $\langle \cos k_s \Delta L \rangle = \langle \cos k_i \Delta L \rangle = \langle \cos(k_s - k_i) \Delta L \rangle = 0$

$$\Rightarrow G^{(2)} \propto 1 + \frac{1}{2} \cos k_p \Delta L$$

classical
stochastic model

\Rightarrow 50% visibility is consistent with classical light

Many future experiments showed higher visibility and violated classical model

Important series of experiments: L. Mandel

See Scully + Zubairy Chapt. 21