Physics 581: Quantum Optics Extra Credit
Distributed: Apr 24, 2014, Due: May 8, 2014

In this project you will explore the ideas and tools of quantum trajectories through the Quantum Monte Carlo Wave Function (QMCWF) simulations discussed in class and the notes entitled "Density Matrices and the Quantum Monte-Carlo Method in Quantum Optics" by Klaus Mølmer. Other references include:

- Class Lectures 11-13.
- R. Dum, P. Zoller, and H. Ritsch, Phys. Rev. A **45**, 4879 (1992).
- K. Mølmer, Y. Castin, and J. Dalibard J. Opt. Soc. Am B, **10**, 524 (1993).
- M. B. Plenio and P. L. Knight, Rev. Mod. Phys. **70**, 101 (1998).

The project involves numerical simulations. You are free to use any computer language/package you feel comfortable with. If you do not have access to such a package, I can direct you to sources here at UNM.
In addition to written answers and figures, please submit all of your computer code.

Problem 1: Damped Rabi Flopping of a Two-Level Atom (30 points)
Consider a two-level atom driven on resonance, $\Delta = 0$, with Rabi frequency $\Omega = 3\Gamma$. The master equation in Lindblad form is

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\left[H_{eff},\rho\right]' + \Gamma\sigma_{-}\rho\sigma_{+}, \quad \text{where } H_{eff} = -i\frac{\hbar\Gamma}{2}|e\rangle\langle e| - \frac{\hbar\Omega}{2}\left(\sigma_{-}+\sigma_{+}\right).$$

(a) Write a computer program to implement the QMCWF algorithm as detailed by Mølmer. Use your algorithm to reproduce Fig. 3 in his notes. Include the error bars as discussed in Sec. 3.3.
(b) Modify your code so that instead of checking for a "jump/no-jump" condition at each interval δt, you simulate the wait-time to the next jump, as discussed in Lecture #12 and by Dum *et al* in the PRA referenced above. Reproduce the results from (a).
(c) *Extra Credit* -- Follow Mølmer's prescription to calculation the spectrum of resonance fluorescence and reproduce the Mollow triplet shown in Fig. 9 of his notes.

Problem 2: Coherent Population Trapping (30 points)

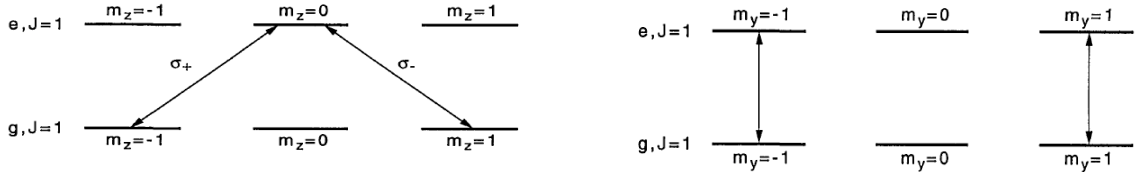
Consider an atom with degenerate Zeeman sublevels in its ground and excited manifold. Take both the ground and excited states to have total angular momentum $J_g = J_e = 1$ so that each manifold has three sublevels $M = -1,0,1$. The atom is driven by laser light on resonance, propagating in the *z*-direction, and linearly polarized in the *y*-direction. The master equation describing the evolution of the atom including spontaneous emission is as given in class,

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\left[H_{eff},\rho\right]' + \Gamma \sum_{q=-1,0,1} D_q \rho D_q^\dagger$$

where the Lindblad operators are: $D_q^\dagger = \sum_{M_e,M_g} \left\langle J_e M_e \middle| J_g M_g; 1q \right\rangle \left| J_e M_e \right\rangle \left\langle J_g M_g \right|$.

The effective Hamiltonian is $H_{eff} = H - i\frac{\hbar\Gamma}{2}\hat{P}_e$, where $\hat{P}_e = \sum_{M_e=-1,0,1} \left| e; J_e, M_e \right\rangle \left\langle e; J_e, M_e \right|$ is the projector onto the excited manifold and $H = -\frac{\hbar\Omega}{2}\left(D_y + D_y^\dagger\right)$ is the Hamiltonian for Rabi frequency Ω. Here $D_y^\dagger = \sum_{q=-1,0,1} D_q^\dagger \mathbf{e}_q \cdot \mathbf{e}_y$, with the "spherical basis" is defined $\mathbf{e}_0 = \mathbf{e}_z, \mathbf{e}_{\pm 1} = \mp\left(\frac{\mathbf{e}_x \pm i\mathbf{e}_y}{\sqrt{2}}\right)$.

As discussed in Mølmer Sec. 4.2, there are two very different pictures of evolution depending upon whether we choose the basis states $\left|J, M\right\rangle$ for *M* with respect to the quantization axis along *z* or along *y*.



(a) Find the dark state as the zero-eigenvalue eigenvector of *H*.
Express the state in terms of the two bases $\left|m_z\right\rangle$ and $\left|m_y\right\rangle$.

(b) Explicitly write out H_{eff} and $D_q^\dagger$ in the two bases.

(c) Write a QMCWF algorithm to simulate the decay of the system to the dark state starting in the initial state $\left|g; m_z = -1\right\rangle$. Do this for both bases and corresponding jump-operators. Reproduce Fig. 11 in Mølmer.

Note: The ratio the Rabi frequency to the spontaneous emission rate is not given. You'll need to experiment with that to reproduce the figure.