Continuous Weak Measurement and Nonlinear Dynamics in a Cold Spin Ensemble

Greg A. Smith, Souma Chaudhury, and Poul S. Jessen
Optical Sciences Center, University of Arizona, Tucson, AZ 85721

Andrew Silberfarb and Ivan H. Deutsch
Department of Physics and Astronomy, University of New Mexico, Albuquerque, NM 87131
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A weak continuous quantum measurement of an atomic spin ensemble can be implemented via Faraday rotation of an off-resonance probe beam, and may be used to create and probe nonclassical spin states and dynamics. We show that the probe light shift leads to nonlinearity in the spin dynamics and limits the quantum non-demolition window of the Faraday measurement. Removing the nonlinearity allows a non-perturbing measurement on the much longer timescale set by decoherence. The nonlinear spin Hamiltonian is of interest for studies of quantum chaos and real-time quantum state estimation.

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The process of quantum measurement involves a fundamental tradeoff between information gain and disturbance. In a projective measurement, this backaction is strong enough to collapse the state of the system and disrupt its coherent evolution. In more realistic scenarios, the system is weakly coupled to a probe, which is then measured to gain small amounts of information at the cost of modest perturbation. Continuous versions of this weak measurement scheme are of particular interest in the context of real-time feedback control and the creation and probing of non-classical states and dynamics [1]. Generally, the coupling of a probe to a single quantum system is so weak that the signal carrying information about the system becomes masked by probe noise. The signal-to-noise ratio of the measurement can be improved by coupling the probe to an ensemble of identically prepared systems, while at the same time the backaction on individual ensemble members can be kept low. Of course the many-body system is now described by a collective quantum state, and when the measurement strength is sufficient to resolve the quantum fluctuations associated with a collective observable, backaction will be induced on the collective state and the uncertainty of the measured value can be squeezed [2]. The creation of such quantum correlation has applications in precision measurement and quantum information processing [3].

In this letter we explore the use of the linear Faraday effect to probe the spins in an ensemble of laser cooled Cs atoms [4,5,6]. Our setup employs a probe beam tuned near the $D_2$ transition at 852 nm, whose linear polarization is rotated by an angle proportional to the net spin component along the propagation axis. Measuring the rotation with a shot-noise limited polarimeter provides a weak measurement of the ensemble averaged spin in real time and with high sensitivity. If the sample is optically thick on resonance, the atom-probe coupling becomes strong enough to allow the collective spin to be measured with resolution below the quantum uncertainty of a many-body spin-coherent state, making it possible to generate quantum correlations within the ensemble.

In the limit of large probe detuning, Faraday rotation has been employed as a quantum non-demolition (QND) measurement of the collective spin, and much interest has been focused on its ability to generate spin squeezed states [7,8], to perform sub-shot noise magnetometry [9] and to entangle separated spin ensembles [10]. We show that spin dynamics in the presence of the Faraday probe is dominated by a nonlinear term generated by the atom-probe coupling, which leads to rapid collapse and revival of the mean spin during Larmor precession of an initially spin-coherent state. The nonlinear collapse is generally much faster than the decay of the mean spin due to optical pumping, and therefore determines the QND “window” of the measurement. Furthermore, both the nonlinearity and measurement strength are proportional to the rate of probe photon scattering [4,11], making the integrated backaction independent of probe intensity and detuning. Therefore, unless steps are taken to reduce the nonlinearity it will determine the maximum possible spin squeezing and multiparticle entanglement. We show that it can be effectively cancelled by applying a bias magnetic field at a specific angle with the probe polarization, and that the non-perturbing nature of the measurement can be recovered on the longer timescale set by probe photon scattering.

A general discussion of Faraday measurements in samples of laser cooled alkali atoms can be found in [4], including measurement sensitivity, sensitivity versus photon scattering tradeoffs, and requirements for significant backaction. In the following we consider corrections to the usual assumption of a QND measurement that arise due to the probe-induced light shift at large detuning. The light shift depends on the probe electric field and the atomic tensor polarizability, $\vec{U} = -\vec{E}_p^{(-)} \cdot \vec{\alpha} \cdot \vec{E}_p^{(+)}$, and generally consists of scalar, vector and rank-2 tensor components. For detunings much larger than the excited state hyperfine splitting, $\Delta \gg \Delta_{HF}$, the scalar and vector components scale as $1/\Delta^2$ [12]. Furthermore, the probe polarization $\vec{E}_p$ is linear and thus $\vec{E}_p^{(-)} \times \vec{E}_p^{(+)} = 0$, in
which case the vector component of $\hat{U}$ is identically zero. Keeping terms to leading order in $\Delta_{HF}/\Delta$ then yields a light shift

$$\hat{U} = \frac{2}{3} U_0 \hat{I} + \frac{\beta}{\Delta} U_0 \left( \hat{\varepsilon}_p \cdot \hat{\Phi} \right)^2$$

(1)

where $\beta$ is a numerical constant depending on the atomic species, and where $U_0 = s \Delta/2$ is the light shift of a two-level atom with unit oscillator strength, natural linewidth $\Gamma$ and saturation parameter $s = (\Gamma/2\Delta)^2 (I_p/I_0)$ associated with a probe intensity $I_p$. Introducing the probe photon scattering rate $\gamma_s = s\Gamma$, and substituting the relevant parameters for Cs in the $F = 4$ hyperfine ground state, we obtain $\beta U_0 / (\Delta/\Gamma) \approx 1.2 \gamma_s/\hbar$. Finally, in the presence of a magnetic field $\mathbf{B}$, the single-spin Hamiltonian takes on the form

$$\hat{H} = g_F \mu_B \mathbf{B} \cdot \hat{\Phi} - 1.2 (\gamma_s/\hbar) \left( \hat{\varepsilon}_p \cdot \hat{\Phi} \right)^2$$

(2)

where we have omitted the scalar (spin-independent) part of the light shift. In addition to the familiar Larmor interaction, we see here a nonlinear term that gives rise to nontrivial dynamics beyond simple rotations and leads to the generation of non-classical spin states. This interaction has been studied in the context of the “kicked top”, a standard paradigm for quantum chaos [13], and leads to phenomena such as the alignment-to-orientation conversion that has been seen in polarization spectroscopy [14].

In our case the nonlinear level splitting induces rapid collapse and subsequent revivals of the mean spin of a Larmor precessing spin coherent state. We model this behavior in detail by setting up and numerically solving a master equation for the $\mathrm{Cs}$ hyperfine ground manifold, thereby fully accounting for both the coherent spin dynamics of eq. (2) and for decoherence from all relevant optical pumping channels. Figure 1 shows an example of the calculated expectation value ($F_x$) as a function of time, for parameters typical of our experiment, clearly illustrating both an initial nonlinear Gaussian collapse and the presence of multiple revivals whose amplitudes are limited by decoherence.

The spin dynamics depend not only on the magnitude of the nonlinear term, but also on the relative orientation of the applied field $\mathbf{B}$ and the probe polarization $\hat{\varepsilon}_p$. Consider the geometry used in our experiment and shown in fig. a, where $\mathbf{B}$ lies in the plane orthogonal to the probe propagation at an angle $\theta$ with respect to $\hat{\varepsilon}_p$. Choosing the $y$-axis along $\mathbf{B}$ we then have $\left( \hat{\varepsilon}_p \cdot \hat{\Phi} \right)^2 = (\sin \theta F_x + \cos \theta F_y)^2$. For magnetic fields $B \gg \hbar \gamma_s/g_F \mu_B$ it is appropriate to make a rotating wave approximation, in which case we find an effective nonlinear term

$$\left( \hat{\varepsilon}_p \cdot \hat{\Phi} \right)^2_{\text{RW}} = \left( -\frac{1}{2} \sin^2 \theta + \cos^2 \theta \right) F_y^2$$

(3)

From this we see that the nonlinear collapse and revival occurs twice as fast at $\theta = 0^\circ$ compared to $\theta = 90^\circ$, and that at a certain critical angle, $\theta = \arctan (\sqrt{2}) \approx 54^\circ$, we have $\left( \hat{\varepsilon}_p \cdot \hat{\Phi} \right)^2 = 0$ and the nonlinear behavior disappears. The corresponding changes in dynamical behavior versus probe polarization angle are clearly visible in our master equation calculations. A more thorough discussion of our theoretical modeling of the spin-probe interaction and master equation treatment will be published elsewhere.

Our experimental setup for Faraday measurements is similar to that described in detail in [4]. We begin by preparing a sample of a few million Cs atoms in a standard vapor cell magneto-optic trap, followed by laser cooling first in a 3D optical molasses and then in a 1D near-resonance optical lattice aligned along the probe direction. Finally, the atoms are optically pumped to produce a spin-coherent state within the $F = 4$ ground hyperfine manifold. Our Faraday probe beam is generated by a tunable MOPA laser, spatially filtered by a single-mode optical fiber and passed through a high quality Glan-Laser polarizer before it is used to probe the atomic sample. The probe intensity profile is very close to Gaussian with a $1/e$ radius of $\sim 1.2\text{ mm}$. This is significantly larger than the typical $0.25\text{ mm}$ radius of the atomic cloud, and ensures that the probe light shift
is reasonably uniform across the ensemble. We use an imaging system to select only the part of the probe beam that passes through the cloud, and analyze it with a simple nearly shot-noise limited polarimeter consisting of a polarization beamsplitter and a differential photo detector (fig. 1b). The resulting measurement of the collective spin has a typical sensitivity that falls short of the requirement for backaction by a factor $\sim 100$, according to the estimate in $\text{[3]}$. This puts us safely in the regime of ensemble-averaged single-spin dynamics and backaction-free measurements.

To observe the nonlinear spin dynamics of eq. (2) we rotate the atomic spin coherent state to point along the $x$-axis, and apply the Larmor field along the $y$-axis. These coordinate axes are chosen so $\mathbf{B}$ forms the desired angle $\theta$ with the (space fixed) probe polarization. We then measure the time-dependent Faraday rotation, which in turn gives a measure of the ensemble average $\langle F_y \rangle$. To improve our signal-to-noise-ratio this process is repeated and the measurement record averaged 128 times. A typical result is shown in fig. 1b, clearly demonstrating the rapid collapse and subsequent revival of the Faraday signal in good agreement with our theoretical model. Of particular note is the approximately Gaussian envelope, which is qualitatively different from the exponential envelope seen when the signal decays due to decoherence caused by the scattering of probe photons. The experimentally observed revival amplitude is typically $\sim 65\%$ of that predicted by theory, suggesting perhaps an extra source of photon scattering from e.g. amplified spontaneous emission in the MOPA gain element, or possibly small deviations from the ideal linear probe polarization.

We have carried out measurements of the type in fig. 1b for a wide range of probe intensity and detuning. Figure 2 shows the observed collapse and revival times obtained by fitting the signal with a gaussian envelope, versus the mean time $\tau_s = \gamma_s^{-1}$ between probe photon scattering events. Our data is in good agreement with theory and confirms the expected scaling behavior over nearly two orders of magnitude in $\tau_s$. Note that $\tau_s$ for an experimental data point must be calculated from the probe intensity at the atomic sample and therefore is somewhat uncertain. Here and elsewhere in the paper we scale the experimental scattering times within a data set by a common factor to obtain the best fit with theory; in general the inferred values for $\tau_s$ agree to better than $20\%$ with those calculated from an independent estimate of the probe intensity.

The nonlinear collapse of Larmor precession depends critically on the relative angle between the Larmor field and probe polarization. Figure 3 shows $1/e$ times for the initial decay, as a function of $\theta$ for an interval between $0\degree$ and $90\degree$. The data supports each of the two qualitative aspects noted above. First the timescales for nonlinear collapse differ by a factor of two between $\theta = 0\degree$ and $\theta = 90\degree$. Second, the $1/e$ decay time is significantly increased at the critical angle $\theta = 54\degree$, where the decay envelope is exponential rather than Gaussian, signifying that the nonlinear term in the spin Hamiltonian has been reduced to a level where it is no longer visible in the dynamics. Also shown in the figure are the predictions of our theoretical model, which are in good agreement with our data over the full range of the angle $\theta$. It is particularly interesting that the Faraday signal decay time at the critical angle can be increased tenfold over the shortest nonlinear collapse time and at least fourfold over the photon scattering time, leading to a very significant increase in useful measurement time. Figure 3 shows further measured and calculated $1/e$ decay times vs. $\tau_s$ at the critical angle. Both obey the expected linear scaling over a wide interval, with the measured decay times reaching a plateau just below 10 ms, most likely
shift, which leads to rapid collapse and revivals in the arising from a spin-dependent term in the probe light dominated by a nonlinear term in the spin Hamiltonian measurements of the collective spin. The spin dynamics is beam, in the geometry typically used for Faraday measurements of the atomic spins and a far detuned, linearly polarized probe interaction between an ensemble of laser cooled Cs atoms, which hints at an extra source of photon scattering due to dephasing caused by ensemble inhomogeneities.

For scattering times $\tau_s < 1$ ms our data suggests that a near-optimal, i.e. decoherence limited measurement is possible. We note again that the best match between theory and experiment is obtained for experimental photon scattering rates $\sim 20\%$ above the best independent estimate, which hints at an extra source of photon scattering in the experiment.

In summary we have performed a careful study of the interaction between an ensemble of laser cooled Cs atomic spins and a far detuned, linearly polarized probe beam, in the geometry typically used for Faraday measurements of the collective spin. The spin dynamics is dominated by a nonlinear term in the spin Hamiltonian arising from a spin-dependent term in the probe light shift, which leads to rapid collapse and revivals in the expectation value for the individual atomic spins. The initial nonlinear collapse is much faster than the spin polarization decay expected from the scattering of probe photons, and thus the most important factor limiting the QND “window” available in a Faraday measurement. We have demonstrated that the effective nonlinearity can be cancelled in the presence of a bias magnetic field forming a critical angle $\theta \approx 54^\circ$ with the probe polarization, thereby increasing the useful measurement window by a factor of ten. To recover a true QND measurement, one can in principle introduce a second, weak laser field tuned to the $D_1$ transition at 894 nm, where the excited hyperfine splitting is larger and a compensating nonlinear term in the light shift can be achieved without adding significantly to the overall rate of photon scattering. Alternatively, the stronger nonlinearity associated with a Faraday probe on the $D_1$ transition can be used to simultaneously drive and observe interesting coherent spin dynamics. We are currently pursuing this approach in an effort to realize a version of the kicked top based on an atomic spin, and hope to use this system to explore fundamental aspects of quantum chaos such as e.g. hypersensitivity to perturbation. The general ability to design nonlinear dynamics will also allow one to extract information from the Faraday signal that goes beyond the mean value of the spin. This might then be used to implement new types of weak measurement, such as real-time estimation of the spin density matrix.

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