Description of my current research and future plans

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April 19, 2009

The common theme that ties together my research is the exploration of the different perspective on natural phenomena furnished by quantum information theory with the aim of gaining new insight into them. I choose to take the perspective provided by information theory in studying physical phenomena because I believe that information, quantified in a suitable form, has an important role – perhaps as important as that of energy and momentum – in understanding the natural world. In my view ideas from information theory, fundamentals of quantum mechanics and statistical mechanics and other fields have to all come together to construct a comprehensive understanding of quantum information dynamics. The need for such an understanding is made immediate by the spectacular advances in experimental science in recent years.

This write up is an attempt to describe in some detail the little pieces of the broader picture that have captured my attention and imagination. My current research is focused on problems in quantum metrology, the theory of open quantum systems and the characterization of nonclassical correlations in quantum systems. More details on the topics that I have worked on, and am working on, and that, to me, are connected by a common thread are provided in the next section followed by a section on my plans for advancing these lines of research further.

I. PREVIOUS WORK

1. Quantum metrology

Measurements are fundamental to all quantitative theories of the physical world. I am interested in the problem of finding the fundamental and practical limits on the measurement uncertainty in parameter estimation. Typically, measurements lead to the transfer of information about the value of one on more observables associated with a physical system to a probe. The nature of the probe, the properties of the interaction between the probe and measured system and the way in which the information about the measured quantities is extracted from the probe are all factors that have a bearing on the achievable minimum uncertainty in a measurement scheme. Quantum metrology [1–5] refers to measurement schemes in which a quantum system is used as the probe. Giovanetti, Lloyd and Maccone [6] studied quantum metrology using techniques from quantum information theory and showed in simple terms how the classical shot noise limited scaling of $1/\sqrt{n}$ applies to the minimum uncertainty with which a parameter γ can be measured when the quantum probe is made of n-subunits initially in a product state. The parameter is imprinted on to the state of the probe through an interaction Hamiltonian $H_{\rm int} = \gamma H = \gamma \sum_{j=1}^{n} h_j$ between the probe and the measured system. By translating a general metrology protocol into the language of quantum circuits, [6] re-derived in a straightforward manner the $1/n$ scaling for the measurement uncertainty corresponding to an entangled initial state of the probe.

Is the $1/n$ scaling for the measurement uncertainty a fundamental limit on the information that can be extracted in a measurement scheme? It turns out that this is true only if the interaction between the probe and the measured system is of the form $\gamma \sum_{j=1}^{n} h_j$. In [7] it was shown that if the interaction Hamiltonian includes k-body couplings between the systems that make up the probe then a measurement uncertainty that scales as $1/n^k$ can be obtained using an initial entangled state for the probe. The formidable experimental challenges that have to be addressed in order to initialize the quantum probe in an entangled state prompted me and my co-workers to consider the problem of the minimum achievable uncertainty with initial product states for the probe when the interaction includes k-body couplings. We found that the uncertainty scales as $1/n^{k-1/2}$ in this case making metrology protocols with $k=2$ give a better performance than the $1/n$ scaling of conventional Heisenberg limited metrology. These results are described in *Quantum-limited metrology with product states* [8]. We showed how such a generalized quantum metrology protocol can be implemented in a Bose-Einstein condensate in [9].

I investigated the effect of decoherence in two types of quantum metrology protocols; one that uses initial product states and the other that uses entangled "Schrödinger-cat" states of the form $(|0\ldots0\rangle+|1\ldots1\rangle)/\sqrt{2}$ for n-qubit probes. This work was done with Prof. Carlton M. Caves at the University of New Mexico. To make the analysis meaningful, we made reasonable assumptions about the resources that were available to do the measurements. The total time it took to perform the measurement was fixed and so was the rate at which the qubits that make up the probe were made available. If not for these assumptions decoherence can always be made irrelevant by making the measurements arbitrarily fast. The interaction time between the probe qubits and the measured system as well as the number of qubits in each probe were optimized over to obtain theoretical lower bounds on the measurement uncertainty in the presence of decoherence. Our analysis of this problem and our results are presented in [10] and this paper complements [6] by including decoherence in quantum metrology protocols. I also studied the closely related problem of decoherence in clock synchronization protocols in collaboration with S. Boixo, A. Datta and C. M. Caves in [11].

2. Open quantum systems and dynamical maps

Decoherence, which I studied in the context of quantum metrology, is only one of the consequences of the unrestrained influence of the environment on quantum systems. Understanding open quantum dynamics is important in devising means of manipulating and controlling physical systems in order to perform quantum information processing tasks. Linear maps of density matrices [12] furnish a powerful method for describing open dynamics. The accepted wisdom in the theory of open quantum systems is that dynamics must be completely positive (CP). Not completely positive (NCP) evolution has been studied in the past by Pechukas [13] but in my view it has not received the attention it deserves.

With Prof. E. C. G. Sudarshan and Prof. Thomas Jordan I showed, using a two qubit example, that unitary evolution of initially entangled states of the system leads to not completely positive reduced dynamics for one of the qubits. We found that the action of the NCP map on all states of the system qubit that are consistent with the initial specification of the entanglement in the extended system is to take density matrices to density matrices. This leads to a physical interpretation for NCP maps and shows that such maps that are observed in quantum process tomography experiments [14] might have an interpretation in terms of errors in the preparation of the initial state of the system leading to residual entanglement with its environment. My investigations into NCP maps was the topic of my Ph.D. dissertation [15] and the main results are also published in [16]. Further work on generalizing the initial results led to three more publications including a paper entitled *Who's afraid of not completely positive maps?* [17–19].

After convincing myself of the validity of not completely positive maps as a description of processes that may be observed in nature, I explored some of the consequences of the existence of such maps in [20]. We found that implementing NCP maps on one of two qubits in a mixed, entangled state using a third qubit that remains separable from the first two can lead to an increase in entanglement between the first two. We also investigated the question of how much information about the overall two qubit state and the interaction between the two qubits can be extracted by observing the reduced dynamics of one of them in *One qubit almost completely reveals the dynamics of two* [21].

3. Lorentz transformations and quantum information

How do relativistic transformations affect properties like purity and entanglement of a quantum system? The answer to such questions are pertinent for understanding how information, treated as a physical quantity, transforms in a manner that respects the symmetries of nature [22]. It turned out that the language of NCP maps that appeared in the context of open quantum dynamics is useful in answering some of the questions related to quantum information and relativity theory.

In [23], we computed the momentum dependent Wigner rotation of the spin state of a massive spin-1/2 particle with momentum p that is subject to a Lorentz boost and showed that the transformation of the reduced spin density matrix is described by a NCP map. We showed how the purity of the spin state may increase as well as decrease depending on the nature of the boost. This work also showed that NCP maps can appear purely due to kinematics, thereby further weakening the arguments for considering exclusively CP maps as a valid description of open quantum dynamics.

The description of the effect of a Lorentz boost on the spin density matrix in terms of a map allowed us to extend the analysis to two particle states. We looked at how the entanglement in the two particle, reduced, spin density matrix is affected by the transformation. We found that this entanglement may increase or decrease depending on the transformation and that there is no simple combination of the entanglement in the spin and momentum degrees of freedom of the two particle state that is conserved under Lorentz boosts contrary to some speculations along these lines by other authors. These results are described in [24].

4. Quantifying entanglement

Quantum entanglement appears in all three research topics that I have mentioned so far. Detecting and quantifying entanglement is a challenging problem in quantum information theory [25, 26]. Measures of entanglement can be broadly divided into two classes. Computationally operational measures are easy to calculate for an arbitrary quantum state and are typically based on the action of positive but not completely positive maps on the density matrix corresponding to the state. Such measures usually lack useful physical interpretation. Measures of entanglement that have clear physical meanings like the entanglement of formation and concurrence are usually hard to compute for arbitrary mixed states because the computation involves an optimization over all possible pure state decompositions of the given mixed state. Such measures of entanglement may be called computationally nonoperational.

With A. Datta, S. T. Flammia and C. M. Caves, I worked on placing lower bounds on nonoperational measures of entanglement using multiple operational measures as constraints. We were able to obtain such bounds for the entanglement of formation, the tangle and the concurrence using the Peres negativity [27] and a new operational measure constructed out of an entanglement detection criterion discovered by Breuer [28] as constraints. Our results were published in [29]. This paper essentially provided a look up table for the minimum possible value of the nonoperational measures of entanglement given the values of the two operational measures for any given state.

A different sort of separability criterion is required for continuous variable systems like the states of the electromagnetic field. In [30], with Manko et. al, I revisited the separability criterion for states of such system constructed by Simon [31] using a partial time reversal transformation. We were able to obtain Simon's criterion using a different approach that also resulted in a continuous family of related separability criteria.

5. Quantum discord

Does entanglement have an essential role to play in making quantum information processing superior to classical information processing in performing certain tasks? It is known that entanglement is needed for quantum super-dense coding and teleportation while for pure state quantum computation, Jozsa and Linden showed that there is a relationship between the spread of entanglement in the system performing the computation and the speedup obtained relative to a classical information processor [32]. On the other hand, for mixed state quantum computation there is evidence that entanglement might not have a significant role to play [33].

Entanglement is not the only kind of nonclassical correlation that may be present in a quantum state. Quantum discord, introduced by Ollivier and Zurek and independently by Henderson and Vedral [34, 35], is an information theoretic measure of all nonclassical correlations, including entanglement in a quantum state. With A. Datta and C. M. Caves, I investigated the quantum discord in the DQC1 model constructed by Knill and Laflamme [36] for mixed state quantum computation. We found that even if there is no detectable entanglement, there is a finite amount of discord in the state of the system at the end of the computation. This suggests that nonclassical correlations other than entanglement might also play a role in enhancing the performance of quantum information processors relative to their classical counterparts. This work led to a paper entitled *Quantum discord and the power of one qubit* [37].

With C. Rodriguez and others, I found that discord can be used to characterize the most general class of states of a system and its environment that leads to completely positive reduced dynamics for the system when it undergoes unitary evolution. A corollary to this result is that entanglement between the system and its environment is not necessary to induce NCP dynamics on the system, other forms of nonclassical correlations would suffice. Details of these results can be found in [38].

6. Other topics

Here I summarize the work I have done on three areas of research over the past few years. I feel that so far I have not devoted enough attention to them to merit subsections of their own in this write up.

- THE SPIN STATISTICS CONNECTION: My interest in the spin-statistics connection is directed at nonrelativistic proofs of the theorem and exploring the limits of applicability of such proofs. The consequences of the spin-statistics connection are implicitly used in our understanding and ability to manipulate many manifestly non-relativistic aggregations of quantum systems starting from solid-state systems to ions in a trap. A clear understanding of the connection without using the language of relativistic quantum field theory in Pauli's proof [39] could provide further insights into how the consequences of spin and statistics can be used to control such systems in novel ways. Inspired by Schwinger's approach [40–42] to the problem, Prof. E. C. G. Sudarshan and I have come up with a relatively straightforward sequence of steps that establishes the spin statistics connection without using relativistic arguments [43–45].
- THE QUANTUM ZENO EFFECT: The quantum Zeno (QZE) and anti-Zeno (AZE) effects can be used as much as a tool for understanding the nature of quantum measurements as it can be used as an example of an essentially quantum phenomenon. It encapsulates the effects of classical interventions on the evolution of a quantum system. I have constructed and analyzed a model of indirect pre-measurements on an unstable quantum state and showed under what conditions QZE and AZE can occur [46]. With K. Modi, I have analyzed a recent experiment by Fisher et al. [47] in which both Zeno and anti-Zeno effects are observed. We have obtained an alternate, fully solvable theoretical model for these experiments that reproduce the observations [48].
- QUANTUM INFORMATION PROCESSING DEVICES USING QUANTUM WAVEGUIDE NETWORKS: Along with my co-workers, I have shown that it is possible to generate and use the quantum resources of superposition and entanglement in a network of quantum electron-waveguides even in the presence of errors and reflections at the gate elements in the network [49]. This serves as a proof of principle that a quantum information processor could be implemented using such a network.

II. FUTURE WORK

The ideas I have for future research projects come in three flavors. There are short term goals which are essentially extensions of completed or ongoing research projects. Then there are ideas I have been thinking for a while now but have not been able so far to do anything significant about it. Finally there are long term goals that I have set for myself. Looking back at previous research statements I have written, I have confidence that I will be able to accomplish a reasonable fraction of these goals and that I will find new areas of interest to explore that are not listed here.

1. Short term goals

i. In [9] we proposed that the generalized quantum metrology protocol that achieves better than $1/n$ scaling for the measurement uncertainty using initial product states can be implemented in Bose-Einstein condensates. Quite a bit more of theoretical work needs to done to craft a real experiment from this initial proposal. I plan to focus on filling in the missing details so that a detailed experimental proposal can be laid out in the near future that can achieve the enhanced scaling. Finding other suitable physical systems like trapped ions and polar molecules that can be the probe in the quantum metrology protocol outlined in [8] is another project I plan to work on.

Apart from the requirement that the probe and the measured system interact via a nonlinear Hamiltonian, we also require that the interaction depend on a parameter that has some variability in time. If the measured parameter γ is a constant then there is very little justification for the effort required to engineer an interaction Hamiltonian with two body couplings on the probe that leads to the $1/n^{3/2}$ scaling for the measurement uncertainty. One might just as well employ a run-of-the-mill metrology scheme with a separable, or even classical, probe and perform the measurements enough number of times to make the uncertainty arbitrarily low . If γ is not a constant and if we assume that there is a finite amount of time needed to assemble a quantum probe of n units then using the new metrology scheme outlined earlier would be advantageous. The current BEC based proposal has the disadvantage that the quantity that is measured is effectively a constant. Modifying this scheme to measure a time varying parameter of interest is another short term research project.

ii. Can quantum discord be a useful figure of merit in characterizing other quantum information processing tasks that employ mixed quantum states? For instance, in [50] it is shown that separable states can be used to distribute entanglement. In this protocol, entanglement is generated between two qubits using a third qubit that remains separable from the first two at all times. Being separable does not mean the there are no nonclassical correlations between the two qubits and the third one during the process. So it is worthwhile to look at the quantum discord in the three qubit state at different stages of the protocol. A few numerical calculations I have done with A. Datta show that there are indeed nonclassical correlations (and no entanglement) between the first and the third qubits after the first step of the procedure when the two interact. These correlations vanish after the last step when the second and third qubits interact leaving the first and the second entangled.

iii. Given a not completely positive map one can ask the question whether there exists a quantum master equation that leads to the same final state as the application of the finite time map on an arbitrary initial state that is compatible with the not completely positive nature of the map. Formally one can write down a Markovian master equation starting from the map and it differs from the usual Kossakowsi-Lindblad master equation in that it has a new set of negative terms. However, interpreting a Markovian master equation that describes the evolution of one part of an initially entangled bipartite state is problematic. It might be that a non Markovian master equation is more appropriate and natural when there is initial entanglement between the system and its environment because, after all, the correlations that are built up between the two are precisely the reason for the failure of the Markov approximation. I plan to study the properties and consequences of the Markovian master equation derived from not completely positive maps as well as look at non Markovian master equations that can closely shadow the dynamics described by the map at each time.

2. Projects in the works

i. Does entanglement have a role to play in the generalized quantum metrology protocols with nonlinear Hamiltonians and separable initial states for the probe? The Hamiltonian is entangling but for short times the signal generated by measuring the probe in some standard separable basis is closely approximated by the signal that would be generated by a separable state of n subsystems with each subsystem evolving at n times its natural frequency. One way we are trying to show that entanglement is not the key behind the performance of these protocols is by computing Meyer-Wallach type measures of entanglement for the state of the probe to show that the value is small. Another approach we are looking into involves constructing a sequential protocol that is equivalent to our measurement scheme in which the probe is made of only a few subsystems. In the sequential scheme the probe will have to interact with the measured system several times but there may not be any entanglement in the probe. This would be obviously so if the probe is made of a single system.

ii. What is the operational meaning of quantum discord in quantum information theory? Does it quantify the resources required to perform an information processing task? Devetak and Winter have shown that one of the two quantities that go into the definition of discord quantifies the distillable common randomness from a bipartite quantum state shared by two parties [51]. In this case the communication between the two parties is assumed to be classical. The other term in the definition of discord is just the quantum mutual information, one half of which quantifies the distillable entanglement from a shared bipartite quantum state. Since, using super-dense coding, perfectly entangled two qubit states can be used to transmit two bits of classical information, it seems that the first term is also related to the distillable common randomness between the two parties, but now using a quantum communication channel. So it seems that discord has an operational interpretation in terms of the difference in the distillable common randomness from a shared quantum state between two parties using a quantum channel as opposed to a classical channel. Shared randomness is a classical resource.

3. Long term plans and new directions

I have a few ideas about the directions my research might take in the long run that I will try to describe here. The contents of this list might change substantially and rapidly without notice for many reasons I can think of. For one, I do plan to take an interest and explore fields in Physics that I am not familiar with at present. Secondly, some of the projects in this list might actually get done or get transferred to the list of short term goals.

i. Even if the $1/n$ scaling of the measurement uncertainty in parameter estimation can be surpassed using initial product states for the probe and nonlinear Hamiltonians, an initial entangled cat state for the probe can still lead to an improvement in the scaling by a factor of $1/\sqrt{n}$. The quantum Cramér-Rao bound says that the measurement uncertainty is inversely proportional to the distance moved by the state of the probe in Hilbert space corresponding to a small change in the parameter. It is not well understood how the entanglement in the initial state of the probe makes it slightly more sensitive to small changes in the parameter than the initial product state. Is there a middle ground in which the same scaling of the measurement uncertainty as the cat state can be achieved using a state that is entangled but at the same time not as difficult to generate as the cat state? What are the theoretical bounds on the minimum uncertainty that can be achieved with generalized quantum metrology in the presence of decoherence? The limits of what can be done in practice with specific choices of projective measurements on the final state of the probe has been explored but absolute bounds coming from the Cramér-Rao bound are not known. A related problem I am interested in studying is multi-parameter estimation using quantum probes. Further, how can one generalize multi-parameter estimation to the case where the interaction Hamiltonian has k-body couplings? These are all pieces of the intriguing puzzle that is the interplay between available resources, their optimal use and the information that can be extracted from a physical system by putting to it the right questions and analyzing the answers in the best possible way.

ii. Recently Shabani and Lidar have shown that considering a wider class of maps on density matrices, including not completely positive ones, as describing errors in a quantum code leads to generalized Knill-Laflamme conditions in the theory of quantum error correction [52]. It has also been shown that entanglement can be used to enhance the performance of error correcting codes [53]. Quantum error correction is a new area in which I can apply the results I have regarding not completely positive maps. What are the consequences of having a limited set of states of the system of interest that are compatible with the entanglement in the overall system? What do inverses of not completely maps look like and can they be implemented in order to correct for errors generated by residual entanglement present between the physical qubits used to encode data and their environment due to inaccurate initialization? I believe that moving beyond the paradigm of completely positive maps in the theory of open quantum systems offers several exciting prospects for new physical insight as well as practical applications.

iii. Experimental advances that have led to the creation of multimode, nonGaussian, continuous variable states have made the need for a comprehensive measure of entanglement in such states essential. The separability criterion discussed earlier is a start but much more needs to be done since it is, for one, necessary and sufficient only for Gaussian states and second, it is only a criterion for separability and not quite a quantitative measure of entanglement. Shchukin and Vogel have proposed a stronger criterion based on higher moments of the state [54]. I am interested in looking at the moment generating functions of nonGaussian entangled states and their partial time reversals in order to search for a criterion that takes into account all the moments of the distribution and hence might be necessary and sufficient for a larger class of states. The next step would be to construct from this criterion a quantitative measure of entanglement.

iv. In his celebrated series of Lectures, Feynman apologizes for not being able to give a simple explanation of the proof of the spin-statistics connection. I plan to continue working on simplifying the nonrelativistic version of the proof that I have refined from the version due to Sudarshan. There are a few caveats in the current version of the proof that needs to be avoided and quite a few common questions for which I need easier answers to. Continuing to work on the spin-statistics connection also helps me keep up an active interest in quantum field theory and other topics in high energy physics that I have learned myself, from courses and by association with people who work in this subject The relationship between quantum information theory and quantum field theory is after all an area of research that is barely starting to open up and potentially holds a wealth of surprises.

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