1.5 (10 points) Challenge problem (a). A hollow cylinder of mass $M$ and radius $R$ rolls without slipping on a table. A particle of mass $m$, attached to the cylinder, slides without friction on the interior of the cylinder.

![Diagram of a hollow cylinder with a particle sliding inside](image)

(a) Define a set of independent generalized coordinates for this system, and give the Lagrangian $L$ in these co-ordinates. (You don’t have to worry about motion in the $y$ direction.)

(b) Give two independent conserved quantities for this system.

(c) Suppose that the cylinder is initially at rest and that the particle is initially at rest at the top of the cylinder. What is the speed $v$ of the particle, relative to inertial space, when it reaches the bottom of the cylinder? The particle must be given a little nudge to get things started away from the initial unstable equilibrium, but the momentum and energy in this nudge can be arbitrarily small and thus can be neglected.