3.4 10 Maximal violation of the CHSH Bell inequality. Consider two qubits, \( P \) and \( Q \). Let \( A = \sigma_P \cdot a \), \( B = \sigma_Q \cdot b \), \( C = \sigma_P \cdot c \), and \( D = \sigma_Q \cdot d \), where \( a \), \( b \), \( c \), and \( d \) are unit vectors in three dimensions. We omit the subscripts \( P \) and \( Q \) on the Pauli operators in the following because ordering in tensor products indicates which system the Pauli operators apply to, but you should feel free to re-introduce these labels whenever it clarifies things. Now let

\[
B = A \otimes B + C \otimes B + C \otimes D - A \otimes D
\]

\[
= \sigma \cdot a \otimes \sigma \cdot (b - d) + \sigma \cdot c \otimes \sigma \cdot (b + d)
\]

\[
= |b - d| \sigma \cdot a \otimes \sigma \cdot f + |b + d| \sigma \cdot c \otimes \sigma \cdot g
\]

be the Bell operator. The quantity we called \( S \) in our discussion of the CHSH inequality is the expectation value of the Bell operator, i.e., \( S = \langle B \rangle \). In the final form of the Bell operator, we introduce unit vectors \( f \) and \( g \), which lie along the directions of \( b - d \) and \( b + d \).

(a) Show that \( |S| = |\langle B \rangle| \leq 2\sqrt{2} \). This result, called T’sirelson’s inequality, determines the maximal violation of the CHSH Bell inequality.

(b) Find the conditions for equality in T’sirelson’s inequality. (Warning: This part is hard, which is probably why it is not included in Nielsen and Chuang’s Problem 2.3, which suggests a less efficient way of proving the T’sirelson bound.)