3.5 10 Consider two systems, $A$ of dimension $d_A$ and $B$ of dimension $d_B$. An arbitrary joint pure state $|\Psi\rangle$, when expanded in an arbitrary product basis $|e_j, f_k\rangle$, looks like

$$|\Psi\rangle = \sum_{j,k} c_{jk} |e_j, f_k\rangle.$$

(a) Show how $|\Psi\rangle$ can be brought into Schmidt form by using the singular-value decomposition of the matrix whose entries are $c_{jk}$, and find the Schmidt vectors for the two systems in terms of the unitary matrices involved in the singular-value decomposition.

(b) Now suppose the two systems have the same dimension $d$. A maximally entangled state of $A$ and $B$ is one such that the marginal density operators are maximally mixed, i.e., $\rho_A = I_A/d$ and $\rho_B = I_B/d$. Find the conditions on $c_{jk}$ such that $|\Psi\rangle$ is maximally entangled, and discuss what this means for the singular values of $c_{jk}$ and thus for the Schmidt coefficients.