3.7 Schmidt-like decomposition for three qubits. Consider three qubits, $A$, $B$, and $C$, each of which has a basis labeled by $|0\rangle$ and $|1\rangle$.

(a) Show that an arbitrary pure state $|\Psi\rangle$ of the three qubits can be transformed to the following Schmidt-like form using local unitary operators on $A$, $B$, and $C$:

\[
\begin{align*}
\cos \theta |0\rangle \otimes \left( \cos \chi |0\rangle \otimes |0\rangle + \sin \chi |1\rangle \otimes |1\rangle \right) & = |\phi_0\rangle \\
+ \sin \theta |1\rangle \otimes \left( \cos \xi (\sin \chi |0\rangle \otimes |0\rangle - \cos \chi |1\rangle \otimes |1\rangle ) + e^{i\delta} \sin \xi (\cos \eta |0\rangle \otimes |1\rangle + \sin \eta |1\rangle \otimes |0\rangle ) \right) & = |\phi_1\rangle
\end{align*}
\]

The states $|\phi_0\rangle$ and $|\phi_1\rangle$ are orthonormal states of $BC$. Five parameters, $\theta$, $\chi$, $\xi$, $\eta$, and $\delta$, are necessary to specify an arbitrary three-qubit pure state; determine the range of these five parameters. [Hint: Schmidt decompose $|\Psi\rangle$ with respect to the division $A$ vs. $BC$. Then Schmidt decompose one of the resulting Schmidt states of $BC$ with respect to the division $B$ vs. $C$, writing the other $BC$ Schmidt state in the resulting Schmidt bases of $B$ and $C$. Use the freedom to rephase states to reduce the number of parameters, and use the freedom to apply local unitaries to get into the standard basis.]

The presence of four terms in $|\phi_1\rangle$, instead of just the first two terms or the last two terms, prevents this from being a genuine three-qubit Schmidt decomposition. This illustrates why there is generally no three-particle Schmidt decomposition.

(b) Find the marginal density operators of the three qubits.