What the #$*! Do We (K)now!? about Quantum Mechanics

Carlton M. Caves
Center for Quantum Information and Control
University of New Mexico

http://info.phys.unm.edu/~caves

View from Cape Hauy
Tasman Peninsula, Tasmania

Aspens in the Sangre de Cristo Range
Northern New Mexico
And that magic is described by quantum mechanics.

There is magic in the world of the very small.

I don't care if you are at Hogwarts, Harry. You can't violate the uncertainty principle. Fifty points from Gryffindor.

Use your quantum mechanics, Harry. Feel the quantum reality.
Quantum mechanics governs the behavior of the very small, but how small are we talking about?

<table>
<thead>
<tr>
<th>Unit</th>
<th>Equivalent</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 meter</td>
<td>m</td>
<td>human height</td>
</tr>
<tr>
<td>(10^{-3}) m</td>
<td>millimeter (mm)</td>
<td>human hair</td>
</tr>
<tr>
<td></td>
<td></td>
<td>size of eukaryotic plant cell</td>
</tr>
<tr>
<td></td>
<td></td>
<td>size of eukaryotic animal cell</td>
</tr>
<tr>
<td>(10^{-6}) m</td>
<td>micrometer ((\mu m))</td>
<td>size of Escherichia coli</td>
</tr>
<tr>
<td></td>
<td></td>
<td>wavelength of visible light</td>
</tr>
<tr>
<td>(10^{-9}) m</td>
<td>nanometer (nm)</td>
<td>thickness of cell membrane</td>
</tr>
<tr>
<td></td>
<td></td>
<td>size of amino acid, X-ray wavelength</td>
</tr>
<tr>
<td></td>
<td></td>
<td>size of an atom</td>
</tr>
<tr>
<td>(10^{-12}) m</td>
<td>picometer (pm)</td>
<td>gamma-ray wavelength</td>
</tr>
<tr>
<td>(10^{-15}) m</td>
<td>femtometer (fm)</td>
<td>size of atomic nucleus</td>
</tr>
</tbody>
</table>
What’s strange about the behavior of quantum systems?

Waves vs. particles

Indeterminacy

Uncertainty principle
\[ \hbar = 6.6261 \times 10^{-34} \text{ joule-sec} \]

Complementarity
What’s strange about the behavior of quantum systems?

Waves vs. particles

Indeterminacy

Uncertainty principle

\[ h = 6.6261 \times 10^{-34} \text{ joule-sec} \]

Complementarity
What’s strange about the behavior of quantum systems?

\[ h = 6.6261 \times 10^{-34} \text{ joule-sec} \]

Given a proton and an electron, balancing the electron’s (positive) energy of motion with its (negative) electrical binding energy, within the constraints of the uncertainty principle, determines the size of an atom to be about 0.1 nanometer. Putting the mass of the proton in each volume of this size gives the density of ordinary matter.
What’s strange about the behavior of quantum systems?

Entanglement
Quantum correlations

“Great! But I might be more impressed if I had a clue what a correlation is, much less a quantum correlation.”

Erwin Schrödinger (1881-1961)
John S. Bell (1928-1990)
N. David Mermin (1935-)
Get ready! I’m going to try to explain why entanglement is weird, but we’ll need a major detour to get there.
Correlations

Student at Harvard University

Smoking

Drinking lattes

Lung cancer

Politically left

Good teeth
Perfect correlation

Left foot
Cowboy boot
Sandal
Biking shoe
Ski boot

Right foot
Cowboy boot
Sandal
Biking shoe
Ski boot
A correlation is an association between things. It's perfect when the association always occurs.
A correlation game

Sacagawea was a Lemhi Shoshone woman, who accompanied the Lewis and Clark Expedition, acting as an interpreter and guide, in their exploration of the Western United States. She traveled thousands of miles from North Dakota to the Pacific Ocean between 1804 and 1806.

**Sacagawea 1US$ coin**

Sculpture, in Bismarck, North Dakota, of Sacagawea and her baby, Jean-Baptiste Charbonneau.

The face on the coin was modeled on a Shoshone-Bannock woman named Randy’L He-dow Teton, then a student at the University of New Mexico.
A correlation game

Correlations between coins held by Alice and Bob can be used to “teleport” the state of Victor’s coin to Bob.

1. Alice only needs to know that her and Bob’s coins are correlated, not whether they are both heads or both tails.
2. Alice doesn’t need to determine whether Victor’s coin is heads or tails.
3. The message she sends to Bob doesn’t reveal whether Victor’s coin is heads or tails.

Sacagawea 1US$ coin

<table>
<thead>
<tr>
<th>Heads</th>
<th>Tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 Up</td>
<td>1 Down</td>
</tr>
<tr>
<td>Bit</td>
<td>1</td>
</tr>
</tbody>
</table>

0 Bit
1
Correlations can be used to send information without actually sending it.
Polarization of light

No polarized glasses  Polarized glasses

A source of polarized light
Polarization of a single photon (particle of light)

Photon

Photon polarization
Photon polarization

Photon
Photon polarization

Quantum coin
Two-state quantum system
Qubit

Photon

Y polarization

1
0

X Y
s r
Quantum coin
Photon polarization
Qubit

Photon

Quantum rules
1. Only one polarization at a time can be prepared or measured.
2. When one polarization is measured, the other is randomized.
Classical bit vs. qubit

A classical bit is either on or off.

- A few electrons on a capacitor
- A pit on a compact disk
- A 0 or 1 on the printed page
- A smoke signal rising from a distant mesa

A qubit has a continuum of on-off properties.

A quantum state tells you the odds for getting 0 or 1 when you examine any of these one-bit coins (linear polarizations).
Planck initiated the study of quantum mechanics when he announced in 1900 the results of his theoretical research into the radiation and absorption of a “black body.”

\[ h = 6.6261 \times 10^{-34} \text{ Joule-sec} \]

Planck’s constant is the scale on which physical phenomena are discrete (or grainy); for example, photons are the expression of the discreteness of the electromagnetic field.
World of classical physics

Continuous, smooth (analogue)

World of quantum physics

Discrete, grainy (digital)

I don’t care if you are at Hogwarts, Harry. You can’t violate the uncertainty principle.

Information-processing perspective

Digital devices (on-off)

Use your quantum mechanics, Harry. Feel the quantum reality.

Continuum of digital properties
A qubit (photon polarization) is a whole bunch of one-bit coins, but we will only need the two coins corresponding to X and Y polarization. A quantum state gives the odds for each of these coins.
Entanglement (at last): quantum correlations

Perfect correlation, with the two results, 00 or 11, being equally likely.

Bell entangled quantum state

\[ \frac{1}{\sqrt{2}}(\ket{00} + \ket{11}) \]

Alice

Bob

Parametric downconverter

X polarization
Entanglement (at last): quantum correlations

Perfect correlation, with the two results, 00 or 11, being equally likely.

Photons A and B are entangled in the Y polarization state.

Alice and Bob measure their photons simultaneously.

From the top of the image, one photon is sent to Alice and the other to Bob.

The images of Alice and Bob are labeled accordingly, with Alice on the left and Bob on the bottom.

The text mentions the parametric downconverter and the Bell entangled quantum state.
A quantum correlation game: Quantum teleportation

Victor

Photon V

Photon A

Bell entangled quantum state

Photon B

Alice makes a special four-outcome polarization measurement on V and A and then sends one of four messages (two bits) to Bob.

Victor runs over to Bob and finds that B now has the same quantum state as V had.

Alice never finds out the quantum state of V, the message doesn't reveal it, and V's initial quantum state is destroyed by the game.
What’s strange about the behavior of quantum systems?

- Entanglement
- Quantum correlations

We cannot account for quantum teleportation in terms of the photons’ having pre-existing polarizations that are discovered by the measurements.

But we haven’t shown that yet. Everything so far can be explained in terms of two 1-bit coins (for X and Y polarizations). But to teleport all polarization, not just X and Y, Alice needs to communicate only two bits to Bob; the correlations of all these coins are too strong to be explained in terms of coins with pre-existing properties.
Quantum entanglement can be used for useful tasks such as teleportation of quantum states.
Greenberger-Horne-Zeilinger (GHZ) entanglement

3-qubit GHZ entangled state: \( \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \)

Let's tell a story: a qubit becomes a two-bit person

\( x=0 \) (heads) is a MAN.
\( x=1 \) (tails) is a WOMAN.
\( y=0 \) (heads) drinks BEER.
\( y=1 \) (tails) drinks WINE.
Ask one person for gender, the other two for drinking preference.

2 heads and 1 tail or 3 tails
2 zeroes and 1 one or 3 ones

\[ x_1 + y_2 + y_3 \text{ is odd.} \]
Ask one person for gender, the other two for drinking preference.

A MAN is always accompanied by two ANTAGONISTS.
A WOMAN is always accompanied by two COMPATRIOTS.

Summarize

2 heads and 1 tail or 3 tails
2 zeroes and 1 one or 3 ones

\[ x_1 + y_2 + y_3 \text{ is odd.} \]
\[ y_1 + x_2 + y_3 \text{ is odd.} \]
\[ y_1 + y_2 + x_3 \text{ is odd.} \]
Ask one person for gender, the other two for drinking preference.

- 2 heads and 1 tail or 3 tails
- 2 zeroes and 1 one or 3 ones

\[ x_1 + y_2 + y_3 \text{ is odd.} \]
\[ y_1 + x_2 + y_3 \text{ is odd.} \]
\[ y_1 + y_2 + x_3 \text{ is odd.} \]

Now ask all three persons for gender.

- 2 heads and 1 tail or 3 tails
- 2 zeroes and 1 one or 3 ones

\[ x_1 + x_2 + x_3 \text{ is odd.} \]

There must be two MEN and a WOMAN or three WOMEN.
A MAN is always accompanied by two ANTAGONISTS.
A WOMAN is always accompanied by two COMPATRIOTS.

Here's why

A MAN is always accompanied by two ANTAGONISTS.
A WOMAN is always accompanied by two COMPATRIOTS.

There must be two MEN and a WOMAN or three WOMEN.
Quantum mechanics: only what we just showed to be impossible occurs.

\[ x_1 + y_2 + y_3 \text{ is odd.} \]
\[ y_1 + x_2 + y_3 \text{ is odd.} \]
\[ y_1 + y_2 + x_3 \text{ is odd.} \]

2 heads and 1 tail or 3 tails
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2 heads and 1 tail or 3 tails
2 zeroes and 1 one or 3 ones
\[ x_1 + x_2 + x_3 \text{ is odd.} \]

There must be two MEN and a WOMAN or three WOMEN.
There must be two WOMEN and a MAN or three MEN.

A MAN is always accompanied by two ANTAGONISTS.
A WOMAN is always accompanied by two COMPATRIOTS.
What’s strange about the behavior of quantum systems?

Entanglement
Quantum correlations
Correlations—even perfect correlations—that violate the rules of ordinary existence.

We cannot account for the behavior of quantum systems by imagining that properties have pre-existing values that are discovered by observation.
Objects in the world of the very small violate the rules of ordinary existence: either they are not individual objects, or they do not have realistic properties. The result is magic.
It’s not only dogs that can’t understand quantum mechanics.

Quantum physics leads you to a world of magic, beyond your imagination, but provides a set of rules to manage and control the magic.

Quantum information science is the discipline that explores information processing within the quantum world, where the mundane constraints of realism and determinism no longer apply.
The more magic you attempt, the harder it gets.

Why not extend the magic into the world of the everyday?

I don't care if you are at Hogwarts, Harry. You can't build a quantum computer. Fifty points from Gryffindor.

Use your quantum mechanics, Harry. A quantum computer would be real magic.

The more magic you attempt, the harder it gets.
Quantum information science.
Engineering the magic.

Truchas from East Pecos Baldy
Sangre de Cristo Range
Northern New Mexico
Private communication

Alice and Bob share a one-time pad (secret random key).

But where do Alice and Bob get the key?
Quantum key distribution using entanglement

Photon A
- Bell entangled quantum state

Alice measures X, Y, U, or V polarization.

Photon B

Alice and Bob share the type of measurement they did, not the result. If they measured the same thing, they keep the shared random bit for their key. Otherwise, they use the results to verify a Bell inequality, which can be used to rule out the presence of an eavesdropper.

Bob measures X, Y, U, or V polarization.
Quantum key distribution using entanglement

Why is quantum key distribution secure?

An unmeasured qubit has no bit value waiting to be discovered. Alice and Bob create the key by measuring the polarizations. Before that, there is no key for an eavesdropper to steal.

“There is no there there.”
Gertrude Stein damning her native Oakland and inadvertently describing quantum systems.

Essential ingredient: Entanglement between qubits