Physics 522, Spring 2016
Problem Set #8
Due: Thursday Mar. 31, 2016 @ 5PM

Problem 1: Addition of spin and orbital angular momentum (15 Points)
Consider an electron with orbital angular momentum quantum number \( l = 1 \) and spin quantum number \( s = 1/2 \).
(a) Using the brute force diagonalization method discussed in Lecture find the simultaneous eigenvectors of \( j^2, \hat{j}_z, \hat{s}^2, \hat{l}^2 \).

(b) Check your answer by using the Clebsch-Gordan coefficients.

(c) Explicitly calculate the matrix elements of \( \hat{l} \cdot \hat{s} \) in the coupled basis. Show that this what you expect based on \( j^2 = \hat{l}^2 + \hat{s}^2 + 2\hat{l} \cdot \hat{s} \).

Problem 2: Stark Shift in Hydrogen (20 points)
Excluding nuclear spin, the \( n=2 \) manifold in Hydrogen has the configuration:

\[
\begin{array}{c}
2s_{1/2} \\
\Delta E_{\text{Lamb}} \\
2p_{1/2}
\end{array}
\quad \quad \quad
\begin{array}{c}
\Delta E_{FS} \\
2p_{3/2}
\end{array}
\]

where \( \Delta E_{FS}/\hbar=10 \text{ GHz} \) (the fine structure splitting) and \( \Delta E_{\text{Lamb}}/\hbar=1 \text{ GHz} \) (the Lamb shift – an effect of quantum fluctuations of the electromagnetic field). In class we neglected these shifts when calculating the Stark shift. This was valid if \( ea_0 E_z \gg \Delta E \).

Let \( x \equiv ea_0 E_z \).

(a) Suppose \( x < \Delta E_{\text{Lamb}} \), but \( x \ll \Delta E_{FS} \). Then we need only consider the \( (2s_{1/2}, 2p_{3/2}) \) subspace in a near degenerate case. Find the new energy eigenvectors and eigenvalues to first order. Are they degenerate? For what value of the field (in volts/cm) is the level separation doubled over the zero field Lamb shift?

(Hint: Use the representation of the fine structure eigenstates in the uncoupled representation)
(b) Now suppose $x > \Delta E_{FS}$. We must include all states ($2s_{1/2}$, $2p_{1/2}$, $2p_{3/2}$) in the near degenerate case. Calculate and plot numerically the eigenvalues as a function of $x$, in the range from $0 \text{ GHz} \leq x < 10 \text{ GHz}$.

(c) Comment on the behavior of these curves. Do they have the expected asymptotic behavior? Find analytically the eigenvectors in the limit $x / \Delta E_{FS} \to \infty$. Show that these are the expected perturbed states.